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Observations of the softening phenomena in the nonlocal cantilever beams

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ABSTRACT

A longstanding puzzle of nonlocal cantilever models is that they do not predict the dynamic softening behaviors of beams compared with the classical beam models. This puzzle exists and is not well solved in the past several years. In this paper, we revisit and make our first attempt to address this issue. By using the weighted residual approaches, the nonclassical force resultants and boundary conditions are obtained. Based on the nonclassical boundary conditions, closed-form solutions for the vibration and buckling problems of the nonlocal Euler–Bernoulli cantilever beams and Timoshenko cantilever beams are derived. Numerical results show that the softening behaviors of cantilever beams can be captured in the nonlocal Euler–Bernoulli beam theory and Timoshenko beam theory. In addition, the differences of the frequencies predicted by the proposed models are increasing larger than those given in the literature as the nonlocal parameter increases, demonstrating clearly the prominent effect of nonclassical boundary conditions on the dynamic behaviors of beams. The asymptotic analysis is constructed to unveil the underlying mechanism of dynamic behaviors of the beams. The numerical results of the analytical solutions obtained in this work may serve as benchmarks for future studies of the dynamic behaviors of composite structures.

1. Introduction

Low dimensional materials and structures in the form of bars, beams, plates and shells have been widely used in sensors, resonators, micro-electro-mechanical systems (MEMS) and nano-electro-mechanical systems (NEMS). In these applications, the size-dependent behaviors of materials and structures have been found due to that the characteristic dimensions of these materials and structures are comparable to the internal length scales. Continuum models in the framework of classical continuum theories are not capable of characterizing the size effect due to the lack of the internal length(s), characterizing the underlying microstructure, in the constitutive equations.

Generalized or higher-order continuum theories of elasticity have been extensively used to account for size-dependent behaviors of materials and structures. Examples of these continuum theories include couple stress elastic theory [1–4], strain gradient theory [5], surface elastic theory [6] and nonlocal elastic theory [7]. Numerous studies have demonstrated that the former three elastic theories can deal with size-dependent materials and structures with stiffening behaviors, whereas the last one with softening behaviors. In view of the great challenges in determining the internal length(s) of the materials and structures, nonclassical continuum models including fewer internal length(s) are desirable. Introduction of the internal length(s) into the constitutive equations will certainly encounter complex nonclassical boundary value problems (BVPs) [8]. To simplify and obtain the closed-form solutions of the BVPs, requirement of the fewer boundary conditions is appreciated. Based on the above discussion, the continuum models based on the nonlocal elastic theory developed by Eringen [7] have found wide applications in the analysis of bending, buckling, vibration and wave propagation problems of carbon nanotubes and graphene sheets in the last decade (see the recent review by Arash and Wang [9]). In the nonlocal elastic theory, the stress at a reference point in the body depends not only on the strains at this point but also on strains at all other points of the body. This theory contains the detailed information about the interactions between atoms or molecules by introducing an internal length scale in the constitutive relations. As one significant

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earlier work, Peddieson et al. [10] proposed the one-dimensional constitutive equation of nonlocal theory to study the static bending of the nanobeams. Since then, the topic of bending, buckling, vibration and wave propagation problems attracts increasing attention and relevant works are mainly classified into two directions. The first direction deals with how the nonlocal parameter affects the static and dynamic behaviors of structures. For more details, the interested readers may refer to Challamel and Wang [11], Wang et al. [12], Lu [13], Reddy [14], Wang et al. [15] and Ghannadpour and Mohammadi [16] for beam models in which the effective bending rigidity of nonlocal beams was stiffer than those predicted by the classical beam models, or refer to Hosseini-Hashemi et al. [17] for plates with at least one free edge in which the frequency curves were not as smooth as ones with other combination of boundary conditions. These works directly combine the nonlocal constitutive equation(s) with the equilibrium equation(s) of classical structures, which will result in the incorrect boundary conditions for structures with free ends. In other words, the resultants in the above works are not variational-consistent. The second direction concentrates on the calibration of nonlocal parameter by either comparing with results of the molecular dynamics simulations [18–21] or more recently by making use of the analytical equivalence between the discrete microstructured models and the nonlocal continuum models [22–27].

In the above-mentioned works, the nonclassical boundary conditions are obtained via simply replacing the classical force resultants by the nonclassical force resultants in the equilibrium equations (see Eqs. (4)–(5) and (8)–(9) or Table 1). Even though it will be later found in the paper that the identical results are obtained for Euler–Bernoulli beams with simply supported boundary conditions and doubly clamped boundary conditions, it is not true for the Timoshenko beam cases subjected to either boundary conditions mentioned above. As a matter of fact, the nonclassical boundary conditions used in the literature should be carefully screened to avoid, for example, the obvious counterintuitive stiffening phenomena for cantilever beams [12,14,28–30] and distinctive values of the nonlocal parameter calibrated by Duan et al. [18] for nonlocal Timoshenko cantilever beams. Therefore, derivation of the nonclassical boundary conditions in the nonlocal elasticity is of great significance from mathematical and mechanical points of view. Indeed, the nonclassical boundary conditions for beams [31,32] modeled by nonlocal theories had been derived using the semi-inverse method developed by He [33]. However, the works by Adali [31] and Kucuk et al. [32] on the derivations of the nonclassical boundary conditions did not provide the closed-form solutions for Euler–Bernoulli beam models and Timoshenko beam models, nor presented any asymptotic analysis on the frequency equation(s). In the literature, the bending solutions were found to be size dependent for beams subjected to concentrated loading. This issue has been addressed in detail by using gradient elastic models that were based on the mixed curvatures in the constitutive equations [11,34] and other variational formulations [35–37]. Lim [38,39] presented a complete and asymptotic representation of the nanobeam model with nonlocal stress via an exact variational principle approach. The nonclassical BVPs of beams were studied and their results of bending deflections showed the fluctuated behaviors. Later, the similar fluctuated behaviors for beam bending and plate bending were also observed [40]. For the buckling problems, the variational formulation had also been studied by Kumar [41]. For the dynamical problems, the distinct frequencies observed in the nonlocal cantilevers had not been well addressed.

The objective of the present work is two folds. First, we use the weighted residual approaches (WRAs) to derive the variational-consistent nonclassical boundary conditions of nonlocal beam models. We then discuss the nonclassical boundary conditions used in the literature. Second, we deal with the BVPs for cantilever beams and subsequently solve the longstanding puzzle of nonlocal cantilever beams. It is believed that the closed-form solutions for the vibration and buckling problems of nonlocal cantilever beams may provide some new aspects for calibrating the Eringen's nonlocal parameter e_0 obtained by other authors [12,14,18,23–26,28,29].

The layout of this paper is as follows. Section 2 briefly reviews the nonlocal elasticity theory. In Section 3, the governing equations of motion of nonlocal Euler–Bernoulli beam and Timoshenko beam models are given. Then, the nonclassical boundary conditions are derived by the WRAs in Section 4. We obtain the closed-form solutions of the frequency and buckling load equations for Euler–Bernoulli beam theory (EBT) and Timoshenko beam theory (TBT) in Sections 5 and 6, respectively. In Section 7, we carry out the asymptotic analysis for the EBT, and summarize the main results in Section 8.

2. Nonlocal theory

In the nonlocal theory of Eringen [7], the nonlocal stress tensor $\boldsymbol{\sigma}$ at point $\mathbf{x}$ is the weighted average of the local stress of all points in the body:

$$\boldsymbol{\sigma} = \int_V K(|\mathbf{x}' - \mathbf{x}|, \alpha)\mathbf{T}(\mathbf{x}')\mathrm{d}\mathbf{x}', \quad \mathbf{T}(\mathbf{x}) = \mathbf{C}(\mathbf{x}) : \varepsilon(\mathbf{x}), \tag{1}$$

where $K(|\mathbf{x}' - \mathbf{x}|, \alpha)$ is the nonlocal modulus or attenuation function incorporating the constitutive equations of the nonlocal effects at the reference points $\mathbf{x}$ produced by the local strain at the source point $\mathbf{x}'$; $|\mathbf{x}' - \mathbf{x}|$ is the Euclidean distance between $\mathbf{x}$ and $\mathbf{x}'$; $\mathbf{T}(\mathbf{x})$ is the classical macroscopic stress tensor at point $\mathbf{x}$, and α is a material constant that depends on internal and external characteristic lengths. $\mathbf{C}(\mathbf{x})$ is the fourth-order elasticity tensor: denotes the 'double-dot product'; $\varepsilon(\mathbf{x})$ is the strain tensor; $\alpha = e_0 a/L_e$ is a small scale parameter where e_0 is a material constant which can be obtained by experiments or through other continuum models; a is an internal characteristic length (e.g., lattice parameter, C–C bond length, granular distance, or the representative volume element of the considered continuum); L_e is an external characteristic length (e.g., crack length, wave length.). As it is difficult to obtain the analytical solution of integral Eq. (1), a simplified differential equation is used to replace the integral constitutive equation in an equivalent differential form as

$$(1 - \alpha^2 L_e^2 \nabla^2)\boldsymbol{\sigma} = \mathbf{T}, \tag{2}$$

where ∇ is the Laplace operator.

3. Governing equations of nonlocal beams

This section reviews the main results of Lu [13] focusing on the governing equations of nonlocal EBT, and Reddy [14] concerning the governing equations of nonlocal TBT for completeness of the content. We consider an elastic beam of length L. The x-axis is chosen along the length of the beam, and z-axis is taken along the thickness of the beam.

3.1. Governing equation of the EBT

The equilibrium equations of motion of the classical EBT are given by

$$\begin{aligned} &Q' - Pw'' = \rho A\ddot{w}, \\ &Q = M', \end{aligned} \tag{3}$$

where $w = w(x,t)$ is the transverse deflection, ρ is the mass density, A is the cross-sectional area of the beam, P is the initial axial force (positive for compression). The prime denotes partial derivatives