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# Spectral stochastic finite element vibration analysis of fiber-reinforced composites with random fiber orientation

K. Sepahvand

*Institute of Mechanics, Universität der Bundeswehr Munich, 85579 Neubiberg, Germany*

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ABSTRACT

Fiber-reinforced composites exhibit random fiber orientations due to the manufacturing tolerances. The present study concerns with a numerical investigation of the vibrational behavior of the long fiber-reinforced composites under uncertainty in fiber orientations. The basic constitutive equations of laminate theory based on the first-order shear deformation are developed in stochastic form. It is assumed that each ply-orientation exhibits individual spatial and random variations from the nominal mean value. These lead to local variations in mechanical properties such as structural stiffness and mass matrices and, accordingly, in structural responses. The finite element formulation utilizes the spectral stochastic discretization in the sense that the random fiber orientations are approximated by a truncated Karhunen–Loève expansion. The expansion provides a framework to capture the spatial and random variations. The generalized polynomial chaos with unknown deterministic coefficients is employed to represent the structural responses. The coefficients are estimated using probabilistic collocation points where any deterministic in-house code or commercial FEM package can be treated as a black-box for the modeling and analysis of structure. A numerical case study is used to illustrate the features of the method, where impact of the uncertainty in fiber orientations on the natural frequencies and mode shapes of a 12 plies rectangular composite plate with nominal orientation of $[0/90]_s$ is investigated.


## 1. Introduction

Fibre-reinforced composites (FRC) are materials made of a polymer matrix, usually epoxy or thermoplastic, together with strong reinforcing fibers mostly made of glass and carbon. The essential high order stiffness to weight and strength ratios make FRC very attractive to be widely used in defense and aerospace technology for many years and more recently in automobiles, wind turbines, medicine, electronic components, sport goods and many other applications. To improve the performance, durability and efficiency of structures and components made of these materials, an exact knowledge of geometrical and material parameters, stacking sequences as well as the fiber orientations is required. Such quantities exhibit significant uncertainties which arise from the variable characteristics of the individual plies, their interconnection and the manufacturing process [1].

Uncertainty modeling in FRC structures has been extensively studied over the past decades, particularly, where deals with material parameter uncertainty. The traditional perturbation method has been used to introduce parameter uncertainty of FRC structures adopting the shape function method [2–4]. The application of the method, however, is limited to the small range of uncertainty. The Monte Carlo (MC) simulation method has received more attention lately to perform static and dynamics analysis of FRC structures with material and fabrication uncertainties [5–7]. The MC methods suffer from low rate of convergence due to their sampling based nature. The first- and the second-order reliability methods (FORM and SORM) have been also applied to analyze the FRC uncertainties [8,9] where the basic random variable concepts employed to model randomness in laminae stiffness properties and material density. The effect of the fiber-orientation angle, elastic modulus and mass density on the vibration properties of the composite plate has been recently investigated in [10]. The paper presents a generic random sampling-based approach for free vibration analysis of angle-ply composite plates. The spatial randomness of fiber orientation makes these methods to be not accurate enough to model random fiber orientation (RFO).

Uncertainty in long fiber orientation having spatial variations is often ignored and most of the developed FRC numerical models assume that fibers are uniformly distributed in the matrix. This is not possible in practical application due to the manufacturing accuracy, fiber misalignment where the orientation of individual

*E-mail address:* sepahvand@unibw.de

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fibers cannot be predicted in a deterministically exact manner [11,12]. The fiber misalignment angle varies throughout the lamina in a random manner and the fiber mean orientation changes spatially so that the laminate mechanical properties are affected from this randomness, and particularly, it leads to variable stiffness [13,14]. Such randomness and misalignments are hard to avoid and will appear due to the manufacturing process and defects, curved surfaces and fiber waviness, as investigated in [15]. The effect of such randomness and variations on the properties and responses of FRC under static loading has been extensively studied in the literature, cf. [1,16–19]. In practice, however, it is not obvious how to model the RFO in numerical simulation, particularly, in finite element method (FEM). One way is to use image analysis, as reported in [20,21] or by using a high resolution X-ray computed tomography system [22]. The results show that the randomness in the fiber orientation is roughly Gaussian in nature. Recently, a micromechanical method has been employed for the prediction of unidirectional composites in which the fiber orientation can possess various statistical misalignment distributions [23]. The method relies on the probability-weighted averaging of the appropriate concentration tensors, which are established by the micromechanical procedure but cannot provide the local information about the RFO. Consequently, the effect of the local variations on the macroscale constitutive parameters has not been captured.

Insofar, as only deflections or vibration behaviors of the FRC structures are required, the macroscale (laminate level) analysis can be performed with no requirement for details of the layer-wise analysis. This is convenient because it allows one to define the aggregate FRC material model with few parameters and the effect of RFO is modeled in macroscale level. The only requirement is to represent the RFO as random field depending on local spatial coordinates and random dimensions. The numerical FEM modeling of such structures, however, requires an adaptive discretization method of RFO when assumed as a random field. The method has to provide high accuracy to approximate the RFO in entire structure while retaining computational efficiency compared to sampling methods.

There are diverse methods for discretization of random fields in the literature, cf. [24]. Among them, the spectral techniques are increasing popular and have the major advantage that the original field is expanded over a complete set of deterministic functions. This provides an opportunity to efficient approximate random fields with rapid decay of the error. Furthermore, the methods are straightforward to be introduced in available deterministic FEM solvers. Two approximation expansions are well-known for random filed discretization by means of spectral methods: the Karhunen–Loève expansion (KLE) and the generalized polynomial chaos (gPC) [25–27]. The methods in combination with the finite element method have been used for numerical investigation of many problems [28–34]. A spectral stochastic finite element formulation with consideration of multi-layer effect and spatial variability of material properties for probabilistic static analysis of laminated composite plates presented in [35]. The study applied the KLE to represent the uncertainty in the elastic material properties, not to the fiber orientation.

The current study, aims to predict the RFO of lamina using spectral representation of random fields. The RFO is represented as random field depending on the spatial and random dimensions. Having the predefined covariance functions for RFO in each direction and each ply, the KLE is utilized to approximate the randomness in each orientation. Owing to the fact that assuming such covariance functions for structural responses is not possible, or at least too difficult, the gPC expansion is devoted to estimate the responses with unknown coefficients. The stochastic FEM formulation is developed for the undamped vibration of the FRC plates with RFO. The major contribution is to represent the spatial and random variations of the RFO as a close form function which can be combined with the deterministic FEM model for stochastic analysis of the FRC structures. Since the deterministic unknown coefficients of structural responses have to be estimated, any deterministic FEM model of the structure can be used to perform the stochastic analysis. To this end, a non-sampling based stochastic analysis is performed in which the deterministic FEM model is operated to realize the structural responses at the collocation points in the random space. The realized estimated responses then are used to calculate the unknown coefficients via an optimization procedure.

This paper is organized as follows: in the next section, we present the basic theory for discretization of RFO. The reformulation of the stochastic constitutive law based on the first-order shear deformation theory is presented in Section 2. The stochastic finite element modeling of the FRC vibrations is demonstrated in Section 4. Numerical results are given in Section 5 and the final section discusses the conclusions.

## 2. Discretization of random fields

It is assumed that the RFO and structural responses can be represented as random fields. In this section, the KLE and gPC expansion are used to discretize the RFO and the responses which allow combining the solution procedure with repetitive runs of an FEM simulations.

### 2.1. Discretization of random fiber orientation

Let $(\Omega, \mathcal{A}, P)$ be a probability space in which RFO can be defined. Here, $\Omega$ denotes the sample space of all the possible outcomes for the orientation, $\mathcal{A}$ is the collection of possible events having well-defined probabilities, and $P$ is probability measure, i.e. the probability of occurrence. The RFO, $\theta(\boldsymbol{x}, \xi)$, in this space depends on spatial coordinates $\boldsymbol{x} \in \mathcal{D} \in \mathbb{R}^d$ and random vector $\boldsymbol{\xi} \in \Omega$, see Fig. 1, i.e.

$$\theta(\boldsymbol{x}, \xi) = \bar{\theta}(\boldsymbol{x}) + \Delta\theta(\boldsymbol{x}, \xi), \quad \theta : \mathcal{D} \times \Omega \to \mathbb{R} \tag{1}$$

where $\bar{\theta}(\boldsymbol{x})$ is the expected nominal fiber orientation and $\Delta\theta(\boldsymbol{x}, \xi)$ denotes the variation due to the randomness. A discretization of such a random field by the KLE method is the approximation of $\Delta\theta$ by means of a finite set of random variables $\boldsymbol{\xi} = \{\xi_1, \xi_2, \ldots, \xi_M\}$. It is assumed that $\xi_i$ are uncorrelated, independent and standard random variables. For instance, $\xi_1$ is defined as standard Gaussian random variable, i.e. $\xi_1 \in \mathcal{N}(0, 1)$ where $\mathcal{N}$ is the standard normal probability density function (PDF). An unidirectional FRC lamina can be modeled as random orthotropic material where the major plane axes ($\zeta_1$ and $\zeta_2$ in Fig. 1) are assumed to be random and serve as the local tangent and normal to fibers, respectively. Let $(x, y, z)$ denote the problem coordinate system used to write the governing equations of the laminate and $(\zeta_1, \zeta_2, z)$ be the local material coordinate of typical lamina. Each lamina is

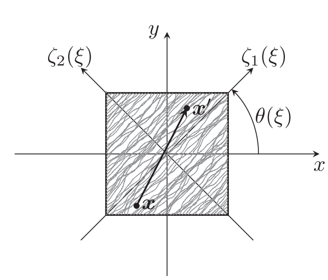


**Fig. 1.** The material coordinates $(\zeta_1, \zeta_2)$ of an unidirectional lamina with random fiber orientation $\theta(\xi)$ in typical element serves as local tangent and normal to the fibers. $\boldsymbol{x}$ and $\boldsymbol{x}'$ denote two arbitrary spatial coordinate points.