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An alternative first-order shear deformation concept and its application to beam, plate and cylindrical shell models

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ABSTRACT

A physical–mathematical interpretation of the alternative first-order shear deformation concept proposed by the author is first presented to get rid of an intuitive aspect of its basic premise that the total deflection w can be assumed as the sum of the bending and transverse shear deflections w_b and w_s. Then, on the basis of several beam and plate illustrative examples, the qualitative theoretical framework of the alternative concept is clarified by comparing with the traditional Timoshenko beam and Mindlin–Reissner plate theories. In addition, a new first-order shear deformation cylindrical shell theory is developed based on the alternative concept and Hamilton's principle to obtain a frequency formula for in-plane vibrations of a thick ring. Finally, the physical–mathematical position of the present theory among the conventional thin-walled structure analysis models is deliberated. The result shows that the present theory is regarded as a refined mathematical generalization of the so-called corrected classical theory and it could lead to a reduction in the number of fundamental variables and governing equations in the modeling of the transverse shear deformable composite structures.

1. Introduction

Moderately thick beams, plates and shells are important structural elements used in many engineering purposes for static deformation, vibration and buckling problems, and there exists a vast amount of transverse shear deformable theories to date, including higher-order theories. However, even though many accurate higher-order theories are available (e.g., Reddy [1,2], Soldatos [3], Hanna and Leissa [4]), first-order shear deformation theories represented by the Timoshenko beam and Mindlin–Reissner plate models still continue to be the focus of much research because of their inherent simplicity, involving recent studies on the advanced microstructure-dependent Timoshenko beam or Mindlin plate models: Ma et al. [5], Roque et al. [6], Abadi and Daneshmehr [7,8], Rahmani and Pedram [9] or Ma, et al. [10].

A recent review is given by Endo [11], in which an overall historical survey on the physical recognition of deformation was carried out, including the so-called corrected classical and first-order shear deformation theories and besides the alternative formulation of first-order shear deformation theories proposed by the author and his co-worker (Endo and Kimura [12]).

The alternative concept is basically equivalent to that of the corrected classical theory in the sense that the total deflection w is superimposed as $w = w_b + w_s$, where w_b and w_s are the bending and transverse shear deflections, respectively, and almost similar relations would be held between them, though their actual methods of formulation are different. Advantages of the alternative deformation concept are summarized as follows (Endo [11]):

(1) Consistent with corrected classical theory in the sense that w_b and w_s are distinguishable physical entities, and are simply added to give the total deflection. Also, both deflections are zero at either clamped or simply-supported ends.
(2) Allows the bending and transverse shear deflections to be obtained concurrently and uniquely using a deductive approach during first-order shear deformation modeling.
(3) Useful for developing an FEM element formulation free from shear locking.

The idea of partitioning the transverse displacements into bending and transverse shear components w_b and w_s was first proposed, to the best of the author's knowledge, by Timoshenko [13], later adopted by Donnell [14], Davidson and Meier [15], Anderson [16], Jacobsen and Ayer [17], Huffington [18], Allen [19], Krishna Murty [20], Senthilnathan et al. [21], Zenkert [22] and recently by Shimpi [23], Senjanovic et al. [24,25], and Thai and his

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co-workers (e.g., [26–28]). Thus, the first-order shear deformation theories based on the concept as $w = w_b + w_s$ are beginning to be presented by several researchers (e.g., Endo and Kimura [12]; Shimpi et al., [29]; Thai and Choi [30,31]; Kolakowski and Krorak [32]).

However, those works all depend on the so-called intuitive techniques in which an idea that w can be superimposed as $w = w_b + w_s$ was introduced *a priori*. On the other hand, with regard to the beam and plate dynamic or static boundary value problems, the author and his co-worker, Kimura, (Endo [11]; Endo and Kimura [12]) carried out the detail numerical verification of the proposed alternative formulation by comparing with the traditional Timoshenko beam and Mindlin plate models for various boundary conditions. Some of those calculated results are shown in the prefaces of Section 3 (for beams) and Section 4 (for plates). Nevertheless, any qualitative theoretical framework of the proposed deformation concept has not been elucidated or clarified completely yet. In addition, to the best of the author's knowledge, no first-order shear deformation cylindrical shell theories based on the alternative concept are known to date.

Considering the above, a physical–mathematical interpretation of the alternative concept is first presented in Section 2 by referring to Vasiliev's works [33,34] on the Reissner static plate theory [35]. Then, in Sections 3 and 4, mainly from the qualitative (not numerical) aspects, the theoretical framework of the alternative first-order shear deformation concept is investigated and clarified on the basis of several simple illustrative examples: statically deformed beams and rectangular plates subjected to some dynamic or static conditions, and by comparing with the traditional Timoshenko beam and Mindlin–Reissner plate theories. And in Section 5, a new first-order shear deformation cylindrical shell theory is developed based on the alternative concept and Hamilton's principle, and then its relational expressions are applied to obtain an approximate formula for in-plane vibrations of a thick ring based on the series-type synthetic-frequency method (Endo and Taniguchi [36,37]). Its results are compared with the authors' previous numerical results (Endo and Taniguchi [38]) based on the conventional first-order shear deformation cylindrical shell theory by Mirsky and Herrmann [39]. Finally, in Section 6, the physical–mathematical position of the alternative first-order shear deformation theory among the conventional thin-walled structure analysis models is deliberated.

All the theoretical models based on the alternative first-order shear deformation concept will be designated as present theory in the coming sections.

2. Physical–mathematical interpretation of the alternative deformation concept

First, consider a basic premise of the alternative concept that the total deflection w is superimposed as $w = w_b + w_s$ by referring to Vasiliev's works [33,34].

According to the Mindlin plate theory [40], three dimensional displacement components u_x, u_y, u_z in a Cartesian co-ordinate system (x, y, z) are assumed as

$$u_x = z\theta_x(x,y,t), \quad u_y = z\theta_y(x,y,t), \quad u_z = w(x,y,t), \tag{1}$$

where t is the time, w is the plate deflection and θ_x, θ_y are the rotations of the plate element in the x–z and y–z planes. The above Eq. (1) is a rather natural assumption as a first step of series-expansion of displacements with respect to the thickness coordinate z.

The equilibrium equations are given by

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = \frac{\rho h^3}{12}\frac{\partial^2\theta_x}{\partial t^2},$$
$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y = \frac{\rho h^3}{12}\frac{\partial^2\theta_y}{\partial t^2},$$
$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = \rho h\frac{\partial^2 w}{\partial t^2}, \tag{2}$$

where ρ is the density, h is the plate thickness, M_x, M_y and M_{xy} are the bending and twisting moments and Q_x, Q_y are the transverse forces, and are expressed as

$$M_x = D\left(\frac{\partial\theta_x}{\partial x} + \nu\frac{\partial\theta_y}{\partial y}\right), \quad M_y = D\left(\frac{\partial\theta_y}{\partial y} + \nu\frac{\partial\theta_x}{\partial x}\right),$$
$$M_{xy} = \frac{1}{2}(1-\nu)D\left(\frac{\partial\theta_x}{\partial y} + \frac{\partial\theta_y}{\partial x}\right), \quad Q_x = k'Gh\left(\theta_x + \frac{\partial w}{\partial x}\right),$$
$$Q_y = k'Gh\left(\theta_y + \frac{\partial w}{\partial y}\right). \tag{3}$$

Here $D = Eh^3/12(1-\nu^2)$ is the plate flexural modulus, E and G are respectively the Young's and shear moduli, ν is the Poisson's ratio, q is the external load and k' is the shear correction factor (i.e., shear coefficient), respectively.

Substitution of Q_x, Q_y in the first and second relations of Eq. (2) into the third expression reduces to

$$D\nabla^2\left(\frac{\partial\theta_x}{\partial x} + \frac{\partial\theta_y}{\partial y}\right) - \frac{\rho h^3}{12}\frac{\partial^2}{\partial t^2}\left(\frac{\partial\theta_x}{\partial x} + \frac{\partial\theta_y}{\partial y}\right) = \rho h\frac{\partial^2 w}{\partial t^2} - q, \tag{4}$$

where ∇^2 is the Laplacian $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$. Here, Eq. (4) allows us to introduce a dilatation potential φ such as (Mindlin [40]; Vasiliev [34])

$$\theta_x = -\frac{\partial\varphi}{\partial x}, \quad \theta_y = -\frac{\partial\varphi}{\partial y}. \tag{5}$$

Substituting Eq. (5) into Eq. (3) and further considering the first and second relations of Eq. (2), we can obtain the following two expressions (Vasiliev [34]):

$$\frac{\partial F_\varphi}{\partial x} = 0, \quad \frac{\partial F_\varphi}{\partial y} = 0, \tag{6}$$

where

$$F_\varphi = \frac{D}{k'Gh}\nabla^2\varphi + w - \varphi - \frac{1}{k'Gh}\frac{\rho h^3}{12}\frac{\partial^2\varphi}{\partial t^2}. \tag{7}$$

From integrated Eq. (6) with respect to x and y, respectively, we arrive at $F_\varphi =$ constant. And further, considering that its constant can be included in φ since we need only derivatives of φ (Vasiliev [34]), we can put as $F_\varphi = 0$, which results in the following expression:

$$w = \varphi + \left[-\frac{D}{k'Gh}\nabla^2\varphi + \frac{1}{k'Gh}\frac{\rho h^3}{12}\frac{\partial^2\varphi}{\partial t^2}\right]. \tag{8}$$

Here, if we consider that as $k'Gh \to \infty$ Eq. (8) reduces to $w = \varphi$, and besides the terms $(-\partial\varphi/\partial x)$ and $(-\partial\varphi/\partial y)$ would contribute to the flexural deformation of a plate via u_x and u_y, it is acceptable to introduce the so-called bending deflection w_b and the definition $\varphi = w_b$. With the above replacement of φ with w_b and taking into account Eqs. (2), (3) and (5), total shearing forces Q_x and Q_y are expressed as

$$Q_x = -\frac{\partial}{\partial x}\left(D\nabla^2 w_b - \frac{\rho h^3}{12}\frac{\partial^2 w_b}{\partial t^2}\right),$$
$$Q_y = -\frac{\partial}{\partial y}\left(D\nabla^2 w_b - \frac{\rho h^3}{12}\frac{\partial^2 w_b}{\partial t^2}\right). \tag{9}$$

While, according to Donnell's method of thinking, we introduce the so-called transverse shear deflection w_s and assume the following relations to hold for ordinary deformation conditions (Donnell [41]):