Composite Structures 146 (2016) 108–113

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Buckling of nanobeams under nonuniform temperature based on nonlocal thermoelasticity

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ARTICLE INFO

Article history:
Received 7 December 2015
Revised 22 February 2016
Accepted 3 March 2016
Available online 9 March 2016

Keywords:
Size effect
Buckling
Nonlocal heat
Nonlocal elasticity

ABSTRACT

Buckling analysis of nanobeam is significantly important for design of nano-electro-mechanical systems (NEMS), especially for those under non-uniform temperature induced commonly by the operation of NEMS. In this work, such issue is investigated with the aids of size-dependent model, i.e. nonlocal thermoelasticity, and size effect of heat conduction on buckling property is considered for the first time. Euler–Bernoulli beam is adopted and reformulated within nonlocal theory. Analytical solution to critical load for various boundary conditions, e.g. SS, CF, CS and CC, is obtained using eigenvalue method. And the temperature distribution is determined using nonlocal heat conductive law. The effect of elastic and thermal nonlocal parameter on the critical load is systematically discussed. It is observed that classical models over-predict the critical load, which may be dangerous in practical applications. The present work is expected to be helpful for design of nanobeam-based devices and systems.

1. Introduction

Progressive advances of nanotechnology have been leading to great development of nano-electro-mechanical systems (NEMS), where high-performance nanostructures, such as: nanowires, nanofilms and nanotubes, are found to be promising building blocks. And these blocks are expected to have wide diversity of applications, e.g. sensors, actuators, transistors, probes, resonators, and etc. As a consequence, the exact characterization of the mechanical properties of nano-structures is significant, of which the buckling property has become one of the topics of primary interest in the following two-folds.

First, due to high aspect ratios, nanobeams that sustain an axial compressive force are especially prone to buckling [1]; second, these structures are often subjected to severe thermal environments during manufacturing and working, and thermal buckling may occur as a result of the induced compressive stresses when nanobeam and nanoplate restrained from in-plane expansion are heated to a specific temperature [2].

It is still a challenge for experimental study of the buckling behavior at the nanoscale due to the difficulties encountered at the current stage. So, theoretical models within continuum mechanics are often adopted for studying the buckling behavior of nano-structures. The classical continuum theory is quite efficient in the mechanical analysis of the macroscopic structures, but it is limited in identification of the size effect on the mechanical behaviors on micro- or nano-scale structures because classical continuum theory does not admit the size dependence. Therefore, to exactly investigate the buckling behavior of nanobeams in thermal environment, size effect of both elasticity and heat conduction should be taken into account.

1.1. Buckling based on nonlocal elasticity

Lots of size-dependent elastic theories are proposed, e.g. nonlocal (or stress gradient) model [3,4], couple stress model [5], strain gradient model [6], and etc., of which nonlocal model has been widely adopted for mechanical analysis at the nanoscale. The works of buckling analysis are as:

For beams: the buckling of a nonlocal shear deformation beam is studied for simply supported boundary condition [7]. Using Ritz method, the bending, buckling and vibration problems of nonlocal Euler beams is solved [8]. Nonlocal elasticity and Timoshenko beam theory are implemented to investigate the stability response of single-walled carbon nanotubes embedded in an elastic medium, where both Winkler-type and Pasternak-type foundation models are adopted [9]. Buckling of various beam theories, i.e. Euler–Bernoulli, Timoshenko, Reddy, and Levinson beam theories, are discussed in the context of nonlocal elasticity [10,11]. Analytical solutions to critical buckling loads are obtained for various beam models under simply supported beams and

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http://dx.doi.org/10.1016/j.compstruct.2016.03.014
0263-8223/

clamped–clamped boundary conditions [12]. There are also works for functionally graded (FG) nanobeams, e.g. buckling of nonlocal FG beams resting on elastic foundation [13], and FG nanobeams based on nonlocal Timoshenko beam [14].

For plates: Levy type solution to buckling of nanoplates using nonlocal elasticity theory is obtained [15]. Shear effects on column buckling of single-layered membranes is investigated [16]. The finite strip method is proposed to study buckling behavior of single and multi-layered graphene sheets including van der Waals effects [17]. Buckling analysis of Mindlin nanoplate [18], third-order shear deformation plate theory [19], double-nanoplate-system [20], variable thickness nanoplates [21], circular graphene sheets [22] are performed within nonlocal elasticity. Buckling of single-layered graphene sheet [23] and double-orthotropic nanoplates [24] embedded in an elastic medium based on nonlocal plate theory is discussed, where Winkler-type and Pasternak-type foundation models are adopted. Subject to linearly varying in-plane load, buckling of orthotropic micro/nanoscale plates via nonlocal continuum mechanics is discussed [25]. More importantly, biaxial buckling behavior of single-layered graphene sheets is predicted using nonlocal plate models, and further proven using molecular dynamics simulations [26].

Note that: buckling analysis of nano-structures have also been done, for example: within couple stress theory [27,28]; surface stress model [29–31], and combination of surface stress and nonlocal model [32,33].

1.2. Thermal effect on buckling

It is known that temperature rise may induce compressive stress in material restrained from axial (in-plane) expansion for beam (plate), as a result, has great effect on buckling behavior of such structures:

Under uniform temperature rise: the critical buckling temperature of a single-walled carbon nanotube [1] and single-walled carbon nanotube embedded in elastic medium [34] are predicted. The effect of temperature on buckling load is evaluated within couple stress elasticity [2], and nonlocal elasticity for multi-walled carbon nanotube [35], as well as combination of surface stress and nonlocal model [36]. An explicit expression is derived for the critical buckling strain for a double-walled carbon nanotube, and the influence of temperature change on the buckling strain is investigated [37]. Beyond that, thermal buckling and post-buckling analysis for moderately thick laminated rectangular plates is conducted for FG materials [38].

Under non-uniform temperature distribution along the height direction: thermal buckling of size dependent Timoshenko FG nanobeams [39], FG rectangular nanoplates based on nonlocal third-order shear deformation theory [40], and nanoplates lying on Winkler–Pasternak elastic substrate medium [41], as well as FG cylindrical shells [42] is studied.

Separately, under longitudinal non-uniform temperature distribution, buckling behavior of elastically restrained steel columns is considered [43]. And under non-uniform temperature distribution in-plane, differential quadrature approach is applied to thermal buckling analysis of cross-ply laminated rectangular plates [44], where the absolute maximum value of the temperature is considered as the index value of the temperature distribution.

It is concluded from above-analysis that: first, buckling of nanobeams and nanoplates has been widely investigated within size-dependent elastic theories. Second, the effect of thermal effect has been considered either under uniform temperature rise or non-uniform one along height direction, and only little works considered the non-uniform temperature along axis (in-plane) for beam (plate). Last, size effect of heat conduction on buckling for nano-structure has never been considered. As a matter of fact, similar to nanomechanics, nanoscale heat conduction has been the hotter research topic from heat transfer in recent years, and its size effect has been systematically studied. For example, different from Fourier heat conductive law, new mechanisms for heat conduction at the nanoscale are proposed, e.g. ballistic–diffusive model [45], collectively diffusive model [46], and etc. Size-dependent heat conductive models are also introduced, i.e. nonlocal model proposed by Guyer and Krumhansl [47] and newly reformulated by Dong et al. [48]. Recently, Yu et al. [49] proposed size-dependent thermoelasticity by combining nonlocal elasticity and nonlocal heat conduction, which will be adopted for buckling analysis of nanobeams subject to non-uniform temperature distribution along axial direction. And the effect of nonlocal parameter of elasticity and heat conduction on the critical load will be discussed.

2. Nonlocal thermoelasticity

Yu et al. [49] systematically review size effect of heat conduction and elastic deformation at the nanoscale, summarize nonclassical model of elasticity and heat conduction, and propose nonlocal thermoelastic model, which may be applicable to thermoelastic analysis for nanoscale issues. The model consists of following equations:

- Equilibrium equations of motion and energy conversation

$$\begin{aligned} &\sigma_{ji,j} + b_i = \rho \ddot{u}_i \\ &q_{i,i} = Q - \rho T_0 \dot{\eta} \end{aligned} \tag{1}$$

- Nonlocal constitutive equation of stress and entropy

$$\begin{aligned} (1-\xi^2\nabla^2)\sigma_{ij} &= 2\mu\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij} - (3\lambda+2\mu)\alpha_\theta\theta\delta_{ij} \\ \rho\eta &= (3\lambda+2\mu)\alpha_\theta\varepsilon_{kk} + \frac{\rho c_E}{T_0}\theta \end{aligned} \tag{2}$$

- Generalized geometric relations

$$\begin{aligned} &\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \\ &(1-\zeta^2\nabla^2)q_i = -k\theta_{,i} \end{aligned} \tag{3}$$

where σ_{ij}, ρ, u_i are stress tensor, mass density, and displacement vector. q_i, T_0, η are heat flux, initial temperature, and entropy. ξ and ζ are nonlocal parameters of elastic and thermal field. λ and μ are Lame coefficients, ε_{ij} denotes strain tensor. θ, k, c_E, α_θ are temperature, heat conductivity, heat capacity, and coefficient of thermal expansion. Q and b_i are heat generation and body force. Finally, ∇^2 denotes Laplace operator. It is noted that if nonlocal parameter of heat and elasticity is omitted, classical thermoelasticity will be obtained. Meanwhile, if deformation is not considered, it will degenerate into nonlocal heat conduction. And nonlocal elasticity is accordingly obtained once the thermal effect is excluded.

3. Buckling analysis of nanobeams

In this section, the buckling behavior of nanobeams will be considered using Euler–Bernoulli beam model as presented in Fig. 1, where the temperature is non-uniform along axial direction. And the width, height and length of the beam are denoted as b, h, and L, respectively. For buckling analysis, dynamical characteristic of nonlocal thermoelastic model will be excluded. As a consequence, entropy constitutive equation makes no sense in static form, and no effect of deformation on temperature will be considered. That is, the nonlocal thermoelastic model mentioned in Section 2 is uncoupled, one may first solve the temperature distribution using 1D nonlocal heat conductive model