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Temperature-dependent nonlinear analysis of shallow shells: A theoretical approach

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ABSTRACT

The paper presents a theoretical formulation for the computation of temperature-dependent nonlinear response of shallow shells with single and double curvatures subjected to transverse mechanical loads while being exposed to through-depth non-uniform heating regimes such as those resulting from a fire. The material nonlinearity arises from taking into consideration the degradation of the material elastic behaviour at elevated temperatures under quasi-static conditions. Two types of boundary conditions are considered, both of which constrain the transverse deflections and allow the rotations about the edge axis to be free. One of the boundary conditions permits lateral translation (laterally unrestrained) and the other one does not (laterally restrained). A number of examples are solved for shallow shells under different types of loading conditions including: an exponential "short hot" fire leading to a high temperature over a relatively short duration; and an exponential "long cool" fire of lower temperature over a longer duration. The limits of the shallow shell equations are investigated through comparison studies. Results show that while current numerical approaches for analysis of laterally restrained shallow shells are often computationally intensive, the proposed approach offers an adequate level of accuracy with a rapid convergence rate for such structures.

1. Introduction

Shallow shells are widely used in the design of lightweight thin-walled structures. They can deliver useful load-carrying capacity by virtue of their curvatures, thereby effectively resisting the externally applied loads with optimum use of materials. The most common applications are in buildings (typically as roof structures), aerospace vehicles, ship hulls, pressure vessels, and car bodies. Most of these structures are potentially at risk of being exposed to severe non-uniform thermal gradients while also externally loaded, such as those resulting from a fire. The consequences of such loading conditions may lead to a reduction of the strength and stiffness of the structure and the development of large deflections, leading to the failure of the structure under the most severe cases. Research in this area has been often focused towards developing efficient shell finite elements (e.g. see Refs. [1–7]). Numerical techniques have been widely used to study the nonlinear behaviour of shells, mainly for their flexibility and ability in dealing with shell problems in comparison with theoretical approaches.

Kumar and Palaninathan [8] employed an eight-node degenerated layered shell element to investigate the geometric nonlinear response of laminated composite cylindrical panels to axial compression and central concentrated load. Their numerical results showed that boundary conditions (BCs) have significant influence on the load-carrying capacity of cylindrical panels so that those with free curved edges and hinged longitudinal edges undergo either limit point or bifurcation failure at very low load levels in comparison with other straight and clamped edge conditions. Using the differential quadrature method, Wang [9] solved the geometric nonlinear buckling problem of thin doubly-curved orthotropic shallow shells with hinged edges. The buckling loads obtained by this method were lower than semi-analytical solutions obtained by either adjacent equilibrium method or partitioned solution method. It was concluded that to ensure the convergence of the solutions, an alternative method should be used as using small load increment increases the computational time required for solving nonlinear problems.

Panda and Singh [10] developed a nonlinear finite element (FE) model to analyse the thermal buckling and post-buckling strength of laminated composite shallow cylindrical/hyperboloid shell panels subjected to uniform temperature rise. In their model, the full nonlinearity effect in the geometry was taken into account in a

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Green–Lagrange sense based on the higher order shear deformation theory. Their numerical results indicated that the post-buckling strength in thermal environment is higher for hyperboloid shell panels in comparison with cylindrical panels. Altekin and Yukseler [11] employed finite difference and Newton–Raphson methods to solve the geometrically nonlinear axisymmetric bending problem of homogeneous and isotropic shallow spherical shells with either clamped or simply supported edges under axisymmetric loads. Their study showed that for partially loaded shells, the influence of the BCs on the central deflection of the shell is negligible. Civalek [12] used discrete singular convolution and differential quadrature methods for the nonlinear static and dynamic analysis of simply supported (laterally unrestrained) and clamped shallow spherical shells resting on elastic foundations. The dynamic analysis of shell structures are well documented in the literature (e.g., see Refs. [13–20]).

Alongside with the advances in numerical techniques, a significant improvement has also been observed in the capability of analytical and semi-analytical approaches to deal with many difficulties in shell nonlinear problems. Development of appropriate theoretical approaches is not only useful for benchmarking FE codes developed for shell-type structures but also for visualising internal structures in composite shells in order to develop much deeper insights into their load-carrying mechanisms. Woo and Meguid [21] studied the nonlinear analysis of simply supported (laterally unrestrained) shallow spherical shells with functionally graded (FG) material properties subjected to transverse mechanical loads and through-depth thermal gradients. The governing equations were established based on the von Kármán theory for large out-of-plane deflections and were solved using series solutions. It was revealed that considering thermo-mechanical coupling effects in the shell formulation can affect the nonlinear response of the shell. Based on the Donnell thin shell theory, van Campen et al. [22] developed semi-analytical methods using the adjacent equilibrium method and partitioned solution method to study the stability behaviour of doubly-curved shallow orthotropic panels under external pressure. In the former method, solutions at the neutral equilibrium position were perturbated to calculate the bifurcation buckling load. However, in the latter method, solutions of the equilibrium and compatibility equations of the panel were perturbated to calculate both the bifurcation buckling and post-buckling solutions that are not confined in the initial post-buckling region. Comparing buckling loads for panels with simply supported and hinged BCs, they showed that the lower buckling load is produced for cases with hinged BCs due to pre-buckling compression.

The nonlinear analysis of an imperfect shallow spherical shell on a Pasternak foundation subjected to uniform loads was presented by Nie [23]. The shell was assumed elastically restrained against rotational, out-of-plane and in-plane displacements. The asymptotic iteration method was applied to obtain an analytical expression for the external load and the central deflection of the shell. Numerical results indicated that imperfections cause a drop in the load-bearing capacity of the shell. Heuer and Ziegler [24] studied the thermal snap-through and snap-buckling of symmetrically layered shallow shells with polygonal planforms and laterally restrained BCs using a two degrees of freedom model derived from a Ritz–Galerkin approximation. Duc and Van Tung [25] studied the nonlinear response of FG cylindrical panels to uniform lateral pressure and uniform and through-depth thermal gradients by an analytical approach associated with a Galerkin method. Formulation was based on the classical shell theory, considering the von Kármán–Donnell-type of kinematic nonlinearity and initial geometrical imperfection. Three BCs were considered in their analysis arising from restriction on the lateral movement of curved and straight edges of the cylindrical panel. Temperature-dependency of material properties of the panel were not taken into consideration. Numerical results revealed that in-plane restraint and temperature conditions play major roles in dictating the response of FG cylindrical panels.

Shen and his colleagues studied the post-buckling behaviour of FG cylindrical shells in thermal environments under an axial compression [26,27], a lateral pressure [28], and a uniform temperature rise over the shell surface and through the shell thickness [29] using the classical shell theory with the von Kármán–Donnell-type of kinematic nonlinearity assumptions. Material properties of the shell were assumed to be temperature-dependent and graded in the thickness direction according to a simple power-law distribution. A boundary layer theory of shell buckling which includes the effects of nonlinear pre-buckling deformations, large deflections in the post-buckling range, and initial geometric imperfections of the shell was used. A singular perturbation technique was employed to obtain the buckling loads and post-buckling equilibrium paths. Yang et al. [30] evaluated the effects of thermal loads, temperature-dependent properties, initial geometric imperfection, volume fraction index, and geometrical parameters on the post-buckling behaviour of FG cylindrical panels with either simply supported (laterally unrestrained) or clamped edges subjected to a combined initial axial force and a uniform temperature change. Their analysis was based on the classical shell theory and the von Kármán–Donnell-type kinematic relations. The critical buckling temperature and the post-buckling temperature-deflection curves were determined using a semi-analytical differential quadrature-Galerkin method associated with an iterative algorithm. They reported that both the buckling temperature and the equilibrium path in the post-buckling regime become lower when the temperature-dependent properties are taken into account in the analysis.

Shahsiah et al. [31] obtained analytical solutions for the thermal instability of FG thin shallow spherical shells based on the Donnell–Mushtari–Vlasov theory. The shell was assumed under three types of thermal loading including a uniform temperature rise, a linear radial temperature, and a nonlinear radial temperature. Girish and Ramachandra [32] presented analytical solutions of the post-buckling problem of symmetric and antisymmetric cross-ply laminated cylindrical shell panels under thermo-mechanical loading based on higher order shear deformation theory. They reported that the shear deformation is less effective on the critical buckling loads of antisymmetric cross-ply shell panels in comparison to symmetric cross-ply shell panels. Nie et al. [33] studied the nonlinear buckling of imperfect orthotropic shallow shells on an elastic foundation using the asymptotic iteration method. Amabili [34] presented the large amplitude of the response of simply supported (laterally not fully unrestrained) doubly-curved shallow shells with rectangular planform to static and dynamic loads. He used the Donnell and Novozhilov shell theories retaining in-plane inertia to obtain the geometrically nonlinear response of the shell. Hamed et al. [35] theoretically and experimentally examined the failure behaviour of thin-walled shallow concrete domes. Their theoretical study included the development of an analytical model for the nonlinear behaviour of materials under failure levels of load, the creep and shrinkage of the concrete material, and the buckling of the dome.

An analytical approach was employed by Bich and Van Tung [36] to study the nonlinear stability of perfect and imperfect FG shallow spherical shells under uniform external pressure with and without considering the effects of uniform and through-depth thermal gradients. In their approach, one term approximation of deflection was used to determine the extremum buckling loads and load–deflection curves for laterally restrained and unrestrained shallow spherical shells. Material properties were assumed to be temperature-independent. Their results showed