Composite Structures 141 (2016) 203–212

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Thermo-mechanical buckling analysis of embedded nanosize FG plates in thermal environments via an inverse cotangential theory

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ARTICLE INFO

Article history:
Available online 15 January 2016

Keywords:
Thermal buckling
Refined theory
FG nanoplate
Elastic foundation

ABSTRACT

In this article, thermal buckling behavior of size-dependent functionally graded nanoplates resting on two-parameter elastic foundation under various types of thermal environments is studied based on a new refined trigonometric shear deformation theory for the first time. It is assumed that the FG nanoplate is exposed to uniform, linear and sinusoidal temperature rises. Mori–Tanaka model is adopted to describe gradually variation of material properties along the plate thickness. Size-dependency of nanosize FG plate is captured by using nonlocal elasticity theory of Eringen. Through Hamilton's principle the governing equations are derived for a refined four-variable shear deformation plate theory and then solved analytically. A variety of examples is presented to indicate the importance of elastic foundation parameters, various temperature fields, nonlocality, material composition, aspect and side-to-thickness ratios on critical buckling temperatures of FG nanoplate. Hence, the present study provides beneficial results for the accurate design of FG nanostructures subjected to various thermo-mechanical loadings.

1. Introduction

Increasing demands for high structural performance requirements, especially in vigorous thermal environments leads to generating a new sort of composite materials known as functionally graded materials (FGMs) which are designed to achieve a functional performance with gradually variable properties in one or more spatial directions. Containing various advantageous properties, FGMs are appropriate for various engineering applications and gained intense interest by several researchers [1,2,4,5,6,7,8]. Moreover, nanoscale plate structures have attracted the interest of some researchers in the field of nanomechanics. Therefore, it is significant to consider the small size influence in the mechanical analysis of nanostructures. Due to the lack of a material length scale, the classical continuum elasticity theory is not capable of describing the size influence. So, size-dependent continuum theories such as nonlocal elasticity theory proposed by Eringen [9,10] are developed to capture the size effects with supposing the stress at a reference point to be a functional of strain of all points of the body. Hence, the nonlocal elasticity theory has extensively applied to analyze the mechanical responses of nanoplates [11,12,13,14,15,16].

Up to now, several theories are developed to analyze the static and vibration behaviors of plates [3]. The simplest theory is known as classical plate theory (CPT) which disregards the influences of shear deformation and hence overestimates natural frequencies of plates. To overcome this problem, many shear deformation plate theories are proposed. The third order shear deformation theory (TSDT) by Reddy [17] which is free from any shear correction factor and satisfies the condition of zero transverse shear strain at the top and bottom of the plate was employed by, Ferreira et al. [18], Oktem et al. [19], and Taj et al. [20].The third order shear deformation theory estimates better results compared to CPT but researchers have obtained more accurate results by adopting various non-polynomial shear deformation theories. In non-polynomial shear deformation theories, the in-plane displacements are the function of thickness coordinate. The function may be trigonometric, exponential or hyperbolic. Zenkour [21] proposed a refined sinusoidal theory for FG plates embedded in elastic medium. Thai and Vo [22] also recommended sinusoidal function but they considered transverse deflection due to bending as well as due to shear. The tangential function was proposed by Mantari et al. [23]. Moreover, Grover et al. [24] implemented a secant function based shear deformation theory for vibration and buckling analysis of composite plates. Also, Neves et al. [25] studied static and vibration behavior of FG plates using a quasi-3D sinusoidal shear deformation theory. The hyperbolic shear strain function was suggested by

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http://dx.doi.org/10.1016/j.compstruct.2016.01.056
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Soldatos [26], Akavci [27] and Mahi and Tounsi [28]. Exponential form of displacement field is proposed by Karama et al. [29] and Mantari et al. [30]; whereas, Aydogdu [31] used logarithmic form of displacement field. Recently, inverse trigonometric shear deformation theory recommended by Sahoo and Singh [32] and Thai et al. [33]. Also, An inverse hyperbolic shear deformation theory for isotropic and FG sandwich plates is proposed by Nguyen et al. [34]. A general assessment of a new inverse trigonometric shear deformation theory for laminated composite and sandwich plates using finite element method is conducted by Grover et al. [35]. Most recently, Kulkarni et al. [36] developed a shear deformation theory namely inverse trigonometric shear deformation theory (ITSDT) for functionally graded macro plates. Therefore, it is innovative to model the smart FG plates especially at nanoscale via a higher order refined theory with only four unknowns.

Referring to the mechanical analysis of size-dependent plate structures, linear free flexural vibration behavior of size dependent functionally graded (FG) nanoplates is investigated by Natarajan et al. [37] using the isogeometric based finite element method. In this work, they applied the nonlocal constitutive relation based on Eringen's differential form of nonlocal elasticity theory. Using an exact analytical approach, free vibration analysis of thick circular/annular FG Mindlin nanoplates is investigated by Hosseini-Hashemi et al. [38]. Using nonlocal TBT and EBT, Şimşek and Yurtcu [39] investigated bending and buckling of FG nanobeam by analytical method. The resonance behaviors of functionally graded micro/nanoplates using Kirchhoff plate theory is studied by Nami and Janghorban [40]. In this study, they adopted the nonlocal elasticity theory and strain gradient theory with one gradient parameter to consider the small scale effects. Daneshmehr and Rajabpoor [41] presented a nonlocal higher order plate theory for stability analysis of FG nanoplates subjected to biaxial in-plane loadings using generalized differential quadrature (GDQ). Based on a modified couple stress theory, a model for sigmoid functionally graded material (S-FGM) nanoplates on elastic medium is developed by Jung et al. [42]. Rahmani and Pedram [43] Analyzed the size effects on the vibration of FG nanobeams based on nonlocal TBT. The nonlinear free vibration of FG nanobeams with fixed ends, i.e. simply supported–simply supported (SS) and simply supported–clamped (SC), using the nonlocal elasticity within the framework of EBT with von kármán type nonlinearity is studied by Nazemnezhad and Hosseini-Hashemi [44]. Bedroud et al. [45] analyzed axisymmetric/asymmetric buckling of moderately thick circular and annular functionally graded (FG) nanoplates under uniform compressive in-plane loads. Also, based on surface elasticity theory Ansari et al. [46] investigated the buckling and vibration responses of nanoplates made of functionally graded materials (FGMs) subjected to a linear thermal loading in pre-buckling domain with considering the effect of surface stress. Also, a nonlocal higher-order shear deformation beam theory for vibration analysis of size-dependent functionally graded nanobeams is present by Ebrahimi and Barati [47]. Zare et al. [48] analyzed the natural frequencies of a functionally graded nanoplate for different combinations of boundary conditions. Thermal buckling can possess a destructive influence on the safety of structures and hence it is regarded as an undesired phenomenon in several studies [1,49]. According to the above-mentioned studies, one can notice that the influences of various temperature environments on buckling behavior of functionally graded nanoplates embedded in elastic foundation is not yet conducted.

In this paper, a new four-variable shear deformation plate theory is developed for the thermo-mechanical buckling analysis of simply supported FG nanoplates on elastic foundation exposed to three kinds of thermal loading. Implementing Hamilton's principle, the nonlocal governing equations are obtained and they are solved via Navier solution method. In the end, the influences of the elastic foundation, different thermal loads, gradient index, nonlocal parameter, aspect and side-to-thickness ratios on the buckling of embedded nanosize FG plates is explored. Some novelties of the present study are stated as follows:

- A new four-variable shear deformation theory is extended for FG nanoplates embedded in elastic foundation containing a cotangential inverse shear strain function without requiring shear correction factors.
- Instead of a classical or power-law model, the material properties of an embedded FG nanoplate in thermal environments are modeled via Mori–Tanaka homogenization scheme, since it is more realistic and provides more accurate results.
- Various types of thermal loading including uniform, linear and sinusoidal temperature rises are considered where this is the first time that sinusoidal temperature change is applied in the analysis of FG nanostructures.

2. Governing equations

2.1. Mori–Tanaka FGM plate model

According to Mori–Tanaka homogenization technique the local effective material properties of the FG nanoplate such as effective local bulk modulus K_e and shear modulus μ_e can be calculated:

$$\frac{K_e - K_m}{K_c - K_m} = \frac{V_c}{1 + V_m (K_c - K_m)/(K_m + 4\mu_m/3)} \tag{1}$$

$$\frac{\mu_e - \mu_m}{\mu_c - \mu_m} = \frac{V_c}{1 + V_m(\mu_c - \mu_m)/[(\mu_m + \mu_m(9K_m + 8\mu_m)/(6(K_m + 2\mu_m))]} \tag{2}$$

where subscripts m and c denote metal and ceramic, respectively and the volume fraction of the ceramic is associated to that of the metal in the following relation:

$$V_c + V_m = 1 \tag{3}$$

The volume fraction of the ceramic constituent of the FG nanoplate is assumed to be given by:

$$V_c = \left(\frac{z}{h} + \frac{1}{2}\right)^P \tag{4}$$

Here p is the gradient index which determines the material distribution through the thickness of the plate and z is the distance from the mid-plane of the FG nanoplate. Therefore, the effective Young's modulus (E), based on Mori–Tanaka scheme can be expressed by:

$$E(z) = \frac{9K_e\mu_e}{3K_e + \mu_e} \tag{5}$$

The thermal expansion coefficient (α) may be expressed by

$$\frac{\alpha_e - \alpha_m}{\alpha_c - \alpha_m} = \frac{\frac{1}{K_e} - \frac{1}{K_m}}{\frac{1}{K_c} - \frac{1}{K_m}} \tag{6}$$

The material composition of FG nanoplate at the upper surface ($z = +h/2$) is supposed to be the pure ceramic and it changes continuously to the opposite side surface ($z = -h/2$) which is pure metal as shown in Fig. 1.

2.2. Kinematic relations

Based on the four-variable shear deformation plate theory, the displacement field at any point of the plate can be written as: