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# Accurate thermo-electro-mechanical buckling of shear deformable piezoelectric fiber-reinforced composite cylindrical shells

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ABSTRACT

An accurate buckling analysis for piezoelectric fiber-reinforced composite (PFRC) cylindrical shells subjected to combined loads comprising compression, external voltage and thermal load is presented in this paper. Based on Reddy's higher-order shear deformation theory, the governing equations for the coupled displacement field and induced piezoelectric field are established. Considering two different kinds of fiber-reinforced configurations, i.e. uniformly distributed (UD) and functionally graded (FG) reinforcements, the buckling solutions of perfect and imperfect PFRC cylindrical shells are obtained by applying separation of variables and Galerkin's method. The influence of geometric parameters, piezoelectric effect, external electric voltage, temperature field and fibers distribution configurations on buckling characteristics and imperfection sensitivity are discussed in detail. The formulation system thus developed is suitable to other shell theories and to account for the coupled electro-magneto-thermo-elastic effects.


## 1. Introduction

Piezoelectric materials possess an advantageous electromechanical interaction property of inducing electric charges in respond to an applied mechanical stress (*direct piezoelectric effect*) or deforming due to an external voltage (*converse piezoelectric effect*). Accordingly, they are excellent candidates for the growing applications of electronic devices such as transducers, pressure sensors and actuators, etc. Nowadays, a great deal of man-made materials that demonstrate piezoelectricity have been found and, among them, piezoelectric ceramics such as barium titanate, lead titanate and lead zirconate titanate, are most commonly used in smart structural systems [1]. However, the inherent brittleness and the limited piezoelectric constants of the monolithic piezoceramics impede their applications as flexible and high-sensitive components in many designs. For this reason, a variety of piezoelectric composites were developed to remedy these deficiencies. Particularly for piezoelectric fiber-reinforced composites (PFRC) that are fabricated by embedding piezoceramic fibers in a polymer matrix [2], the specific mechanical performance and piezoelectric effect can be achieved by adjusting the distribution configuration of the constituent fibers.

These intelligent materials were first examined by Hagood et al. [3] and Wilkie et al. [4]. In an earlier stage, the efforts were mainly devoted in the modeling of effective material properties of these advanced composites [5–8]. A number of micromechanical investigations showed an improved piezoeffect comparing with the single-phase piezoelectric materials. Later, with increasing applications in modern electronic technology, electro-mechanical coupling analysis of such piezoelectric structures started to attract a widespread attention. In many practical engineering systems, research in structural vibration and control has been considerably developed. Pan et al. [9] conducted a theoretical analysis and test on the vibration response of a piezoelectric fiber actuator. By using macro-fiber piezoelectric composites, Yang et al. [10] presented a novel measurement system for harvesting vibration energy. Przybylowicz [11] and Sohn et al. [12] investigated some innovative strategies for vibration control. In other aspects, the elastic deflection and stresses in beams bonded with flexible PFRC sheets were investigated by Shiyekar and Kant [13], Mareishi et al. [14] and Panda et al. [15]. In solid mechanics, buckling and post-buckling analyses always pose many baffling yet classical problems to engineering. In recent years, a lot of theoretical studies have focused on buckling of laminated or functionally graded piezoelectric composite structures [16–19]. By combining the advantages of functionally graded materials (FGMs) and PFRC, Shen [20] contributed a lot on the post-buckling response of FGM cylindrical shells with

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surface-bonded PFRC actuators. In the study, Reddy's higher-order shear deformation theory was employed and the effect of control voltage on buckling was discussed in detail. Subsequently, similar issue was also carried out by Dai and Zheng [21] based on the classical thin-walled shell theory. In the study, response of PFRC layer under the external voltage was simplified into action of a compressive load, and only the displacement components were taken as the basic variables. The change in electric field intensity with respect to structural deformation was ignored in their study.

In a general overview, the previous studies provided very little information on buckling of PFRC structures. To pursue further the mechanical–electrical coupling of PFRC, this paper concentrates on an accurate thermo-electro-elastic buckling analysis for PFRC cylindrical shells. Two types of PZT fiber configurations, i.e. uniformly distributed (UD) and functionally graded (FG) through shell thickness [22], are introduced in the article. Besides, in view of the materials feature, PFRC plates and shells with the larger ratios of thickness to width/radius are generally required for manufacturing electric devices. The importance of transverse stress effects in the thickness direction should be recognized and higher-order theoretical models need to be adopted. In this aspect, Reddy et al. [23–25] proposed a higher-order shear deformation theory, in which a parabolic distribution of transverse shear strains through the shell thickness was followed. The related formulation has been frequently applied in the static, buckling and vibration analysis for composites and piezoelectric structures [26–30]. Later, higher-order theories were furnished by many researchers in the modeling of laminated composite structures [31–33]. Among them, a favorable unified formulation (CUF) presented by Carrera [34–36] was widely adopted in the mechanical and multifield problems of advanced composites and smart structures. Based on the given unified displacement variables, the developed theories and finite element methods were very efficient in dealing with various static and dynamic behaviors of thick-walled plates and shells [37–39]. In this study, by means of the variational principle, the governing equations of electro-elastic cylindrical shells are established using Reddy's higher-order shear deformation theory (HSDT). By separating the variables, analytical solutions are derived for clamped and simply supported PFRC cylindrical shells. Afterwards, using the mode function and Galerkin's method, buckling of shells with the initial geometric imperfections is investigated. Through solving the coupled displacement and electric fields, a detailed parametric analysis is reported to highlight the effects of geometric parameters, fiber reinforcements, material properties, temperature changes, external electric voltage and imperfection sensitivity.

## 2. Modeling and formulation

A PFRC cylindrical shell with coupled compressive load $F$, temperature rise $\Delta T$ and electric potential $\pm V$ on the shell surfaces is considered. As illustrated in Fig. 1, the geometric parameters defined are length $l$, mid-surface radius $R$ and thickness $h$. The coordinate system refers to the middle shell surface and three principal axes ($x$, $y$ and $z$) are along the longitudinal, circumferential and outward normal directions. The corresponding displacements are denoted by $u$, $v$ and $w$, respectively. The PFRC is constructed by embedding continuous parallel piezoelectric fibers into an isotropic matrix. The unidirectional fibers are parallel with the $x$-axis and the electric field is transverse to the fiber direction. According to the fiber layout in the thickness direction, uniformly distribution (UD) and functionally graded distribution (FG) are considered. Meanwhile, two types of FG-PFRC shells are investigated: Type A shell is fiber rich on the outer side ($z = h/2$) and matrix rich on the inner side ($z = -h/2$) while Type B shell is opposite to Type A in fiber configuration.

The effective material proprieties of PFRC are generally obtained either by the micromechanics approach or through experiment. Currently, such theoretical studies are commonly based on the rule of mixture (Voigt model) [6–8] and the Mori–Tanaka model [5]. To conveniently and precisely predict the overall material properties, the constitutive relation is obtained in this paper by studying the strength of materials [8]. In the PFRC body, $v_f$ and $v_m$ represent the volume fractions of fiber and matrix, respectively. According to the distributed fiber configurations, there are

- Uniform distribution (UD):

$$v_f = v^*, v_m = 1 - v_f \tag{1}$$

- Functionally graded distribution (FG):

$$v_f = 2v^*(1/2 \pm z/h)^k, \ v_m = 1 - v_f \tag{2}$$

where the volume fraction $v^*$ ranges from 0 to 0.5. The index $k$ controls the profile of fibers distribution. For $k = 1$, $v^*$ indicates an average volume fraction of fibers. Operators " + " and "−" correspond to Type A and Type B shells, respectively.

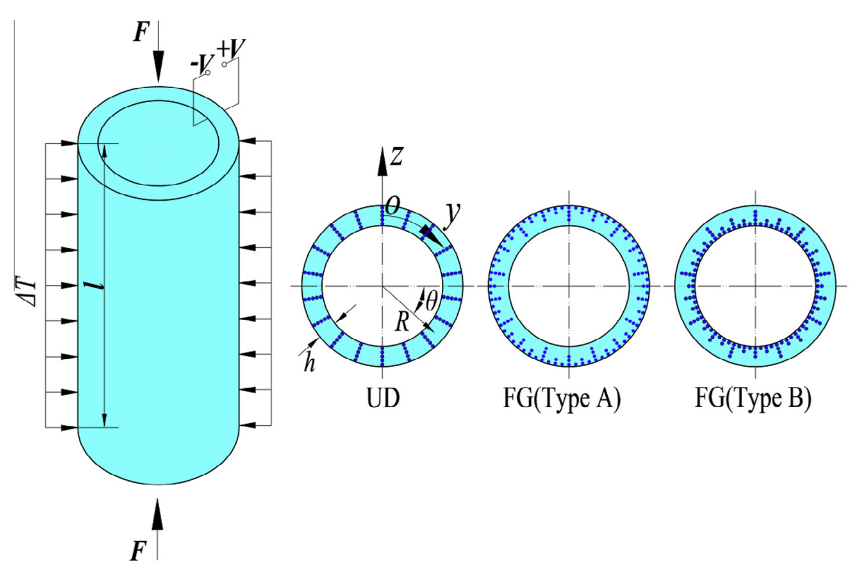


**Fig. 1.** Geometric model of a piezoelectric fiber-reinforced composite (PFRC) cylindrical shell.