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An analytical expression for fundamental frequency of the composite lattice cylindrical shell with clamped edges

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ABSTRACT

A solution of the free vibrations problem formulated for a composite lattice cylindrical shell with clamped edges is presented in this paper. The lattice shell is composed of a large number of helical and hoop ribs and modelled as a continuous orthotropic thin cylinder with effective stiffness parameters. A solution of the equations of motion of the shell is based on the Fourier decomposition and the Galerkin method and yields an analytical formula for the calculation of a fundamental frequency. It is demonstrated that starting from a certain density of the lattice structure the value of fundamental frequency does not depend on the number of helical ribs. This result is verified and confirmed using finite-element analysis. Applications of this formula to the determination of the parameters of lattice structures and design of composite lattice shells with required fundamental frequencies are demonstrated using numerical examples. It is shown that the analytical formula presented in this article provides an efficient tool for rapid calculation of the fundamental frequency which can be used for the assessment of the structural stiffness of the composite lattice shells in the design analysis.

1. Introduction

Lattice cylindrical shells manufactured by filament-winding from composite materials with high elastic moduli have unique specific strength and stiffness characteristics. Due to these properties, the lattice shells are efficiently utilised as interstage adapters and bodies of spacecraft [1,2]. Conventionally, mechanical analysis of lattice cylindrical shells is performed using continuous and/or discrete models. In the continuous model, the lattice cylinder is replaced with the "quasi-equivalent" continuous orthotropic shell having effective stiffness parameters corresponding to the original lattice structure. This approach, due to its relative simplicity, is widely used in the structural analysis and design of composite lattice shells. Continuous models of various popular composite anisogrid lattice structures are considered in the monographs published by Vasiliev [3], and Vasiliev and Morozov [4]. Various analyses of lattice shells based on the continuous models were reported by Totaro and Gurdal [5], Buragohain and Velmurugan [6], Paschero and Hyer [7], Totaro [8,9], Zheng et al. [10].

The discrete models are built using finite-element modelling and normally could be composed of the beam, shell, or solid elements. Results of finite-element analyses of composite lattice shells can be found in the papers published by Hou and Gramoll [11], Zhang et al. [12], Frulloni et al. [13], Fan et al. [14], Morozov et al. [15].

The filament-wound anisogrid composite lattice cylindrical shells utilised in aerospace structures are subjected to time-dependent operational loads. Hence, the dynamic analysis involving a determination of vibration frequencies and modes is an important part of the design process for such structural components. Often, the solutions of dynamic problems related to the determination of the fundamental frequency of the lattice shells are employed at the design stage. Using the value of fundamental frequency, the overall stiffness and mass of the shell structure can be assessed. This is due to the fact that this parameter reflects a combined mutual effect of the bending stiffness and the mass-per-unit-length of the shell wall. To perform such an assessment, multiple analyses are to be completed at the design stage. Thus, it is advantageous to have a compact analytical equation that would provide the value of the fundamental frequency without the need for numerical modelling and analysis. It should be noted that to date not too many studies of dynamic behaviour of composite anisogrid lattice cylindrical shells have been reported in the literature. For instance, an applied method of calculations of the natural frequencies of the composite lattice shells was presented

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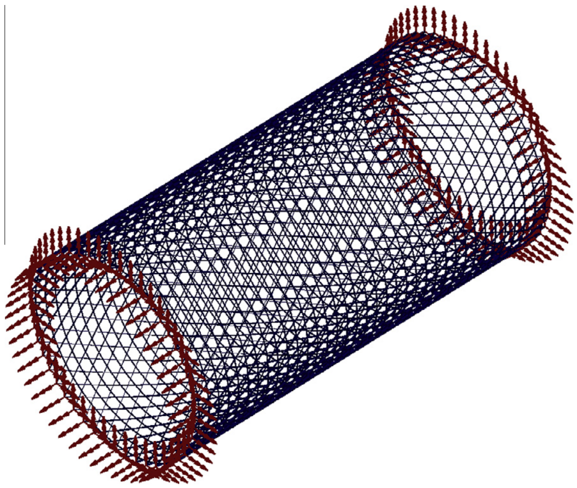

Fig. 1. Clamped–clamped anisogrid lattice cylindrical shell.

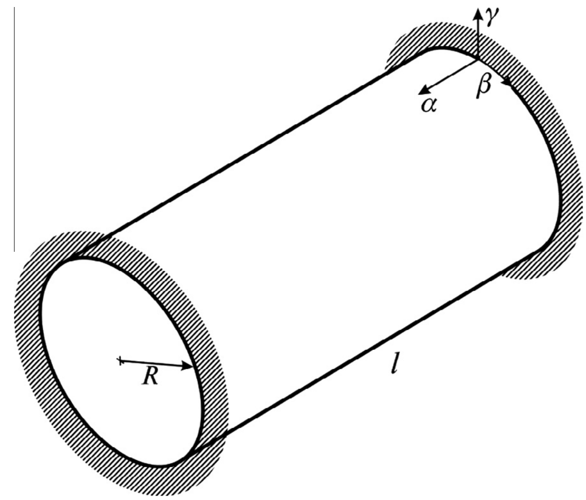

Fig. 2. Clamped–clamped orthotropic continuous cylindrical shell.

by Vasiliev and Skleznev [16] and Skleznev [17]. In their work, the shells were modelled as the thin-walled rods characterised by effective averaged stiffness parameters. Free vibrations of a cantilever composite lattice cylindrical shell with the rigid disk attached to its free end have been analysed by Lopatin et al. [18] using semi-membrane theory of orthotropic cylindrical shells. Nevertheless, a growing popularity of the composite anisogrid lattice cylindrical shells in various aerospace applications calls for further studies of their vibrations. In particular, as mentioned earlier, the determination of the fundamental frequencies of the cylindrical lattice shells with different types of support is imperative in the design analyses. In this article, a compact analytical formula for rapid calculations of the fundamental frequency of the composite lattice cylindrical shell with fully clamped ends is obtained based on the continuous model of orthotropic shell having effective stiffness parameters matching those of the original lattice structure. A solution of the equations of motion of the shell is based on the Fourier decomposition and the Galerkin method. Results of parametric analyses performed for the shells with various numbers of helical ribs show that once a certain density of the lattice structure is reached, further increase in the number of helical ribs does not affect the value of fundamental frequency. This important conclusion has been verified using the finite-element analysis. It is also demonstrated how the structural parameters of the lattice shell can be selected using the aforementioned analytical formula.

2. Governing equations

Consider a composite anisogrid lattice cylindrical shell with both ends clamped as shown in Fig. 1. The lattice structure of the shell is composed of two regular systems of ribs: the helical and hoop ones. It is assumed that the density of the lattice structure (number of ribs per unit length) is high enough to employ a continuous model of the shell. According to this, the lattice shell is replaced with the continuous orthotropic one having equivalent effective stiffnesses. The middle surface of the shell of radius R and length l is referred to the curvilinear coordinate frame $\alpha\beta\gamma$ as shown in Fig. 2. The coordinate axes α and β are directed along the axial and hoop directions, respectively. The axis γ is orthogonal to the middle surface. The motion of the shell is modelled using classical theory of orthotropic cylindrical shells in the following form [3]:

$$
\begin{aligned}
&\frac{\partial N_\alpha}{\partial \alpha}+\frac{\partial N_{\alpha\beta}}{\partial \beta}-B_\rho\frac{\partial^2 u}{\partial t^2}=0\\
&\frac{\partial N_{\alpha\beta}}{\partial \alpha}+\frac{\partial N_\beta}{\partial \beta}+\frac{1}{R}\frac{\partial M_{\alpha\beta}}{\partial \beta}+\frac{1}{R}\frac{\partial M_\beta}{\partial \beta}-B_\rho\frac{\partial^2 v}{\partial t^2}=0\\
&\frac{\partial^2 M_\alpha}{\partial \alpha^2}+2\frac{\partial^2 M_{\alpha\beta}}{\partial\alpha\partial\beta}+\frac{\partial^2 M_\beta}{\partial \beta^2}-\frac{N_\beta}{R}-B_\rho\frac{\partial^2 w}{\partial t^2}=0
\end{aligned}
\tag{1}
$$

in which t is the time, B_ρ is the mass of the shell per unit area, N_α, N_β and $N_{\alpha\beta}$ are the membrane stress resultants; M_α, M_β and $M_{\alpha\beta}$ are the bending and twisting moments (resultant couples); u and v are the displacements of the points of the middle surface along the axes α and β, respectively; w is the deflection in the radial direction.

The equations given by Eq. (1) should be supplemented by constitutive equations, relations between displacements and strains, and boundary conditions. The constitutive equations has the form

$$
\begin{aligned}
&N_\alpha=B_{11}\varepsilon_\alpha+B_{12}\varepsilon_\beta,\quad N_\beta=B_{21}\varepsilon_\alpha+B_{22}\varepsilon_\beta,\quad N_{\alpha\beta}=B_{33}\varepsilon_{\alpha\beta}\\
&M_\alpha=D_{11}\kappa_\alpha+D_{12}\kappa_\beta,\quad M_\beta=D_{21}\kappa_\alpha+D_{22}\kappa_\beta,\quad M_{\alpha\beta}=D_{33}\kappa_{\alpha\beta}
\end{aligned}
\tag{2}
$$

where $\varepsilon_\alpha, \varepsilon_{\alpha\beta}, \varepsilon_\beta$ are the membrane strains of the middle surface; $\kappa_\alpha, \kappa_\beta, \kappa_{\alpha\beta}$ are bending and twisting deformations of the middle surface; $B_{11}, B_{12}, B_{22}, B_{33} (B_{21}=B_{12})$ and $D_{11}, D_{12}, D_{22}, D_{33} (D_{21}=D_{12})$ are the membrane and bending stiffnesses of the shell wall. The strain–displacements relationships are given by

$$
\begin{aligned}
&\varepsilon_\alpha=\frac{\partial u}{\partial \alpha},\quad \varepsilon_\beta=\frac{\partial v}{\partial \beta}+\frac{w}{R},\quad \varepsilon_{\alpha\beta}=\frac{\partial u}{\partial \beta}+\frac{\partial v}{\partial \alpha}\\
&\kappa_\alpha=-\frac{\partial^2 w}{\partial \alpha^2},\quad \kappa_\beta=-\frac{\partial^2 w}{\partial \beta^2}+\frac{1}{R}\frac{\partial v}{\partial \beta},\quad \kappa_{\alpha\beta}=-2\frac{\partial^2 w}{\partial\alpha\partial\beta}+\frac{1}{R}\frac{\partial v}{\partial \alpha}
\end{aligned}
\tag{3}
$$

The system of equations, Eqs. (1)–(3) is of eighth order with respect to the variables α and β. Hence, four boundary conditions are required at each end $\alpha=0$ and $\alpha=l$. In the case under consideration, i.e. for the fully clamped edges, these conditions are

$$
u=0,\quad v=0,\quad w=0,\quad \frac{\partial w}{\partial \alpha}=0 \tag{4}
$$

Substitution of Eqs. (2) and (3) into Eq. (1) yields the following governing equations of motion written in terms of displacements u, v, and deflection w: