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Analysis of thin-walled open-section beams with functionally graded materials

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A B S T R A C T

In this paper, an analytical modeling of thin-walled open-section beams with functionally graded materials (FGMs) is presented, regarding mono-symmetric I- and channel-sections. The mechanical properties of beam such as Young's modulus and shear modulus are assumed to continuously vary in the thickness direction based on the power law distribution of volume fraction of metal or ceramic. The locations of center of gravity and shear center for FG beams are derived. The proposed theory considers restrained warping applicable to the thin-walled FG beam based on Vlasov's assumptions. General governing equations are derived and directly solved, therefore, exact solutions can be achieved. In addition, the effects of gradual law and thickness ratios of ceramic or metal on behavior of various thin-walled FG beams are investigated.

1. Introduction

Functionally graded materials (FGMs) as a new class of advanced composites have been increasingly used over the past few decades in a variety of structures due to a number of their advantages such as high ratios of stiffness and strength to weight, high fracture toughness and enhanced thermal properties. This is due to the fact that the ceramic constituents of FGMs are able to withstand high-temperature environments in spite of their better thermal resistance characteristics, while the metal constituents provide stronger mechanical performance and reduce the possibility of catastrophic fracture. Therefore, the FGMs have gained considerable attention from areas such as aerospace engineering, nuclear technology, civil and mechanical engineering.

As the use of FGMs increases, the structural members made of FGMs which are considered the distribution of volume fractions of constituent phases have been presented over the years [1–7] since the early work of Aboudi et al. [8]. Sankar [9] obtained the solution for FG rectangular beam subjected to a sinusoidal transverse loading. Based on Euler–Bernoulli type beam theory, it was found that the exact elasticity solution for stresses and displacements was valid for long, slender beam if the load was a slowly varying function of the axial coordinate. It also showed that geometrical effects on stress concentration in short or deep beam cannot be treated by the technical theory. Zhu and Sankar [10] also dealt with the similar problem in which Young's modulus was expressed by a polynomial function in the thickness direction. By means of the semi-inverse method, an analytical solution of FG cantilever beam subjected to different loads was derived by Zhong and Yu [11]. The obtained solution was valid for arbitrary graded variations of the material distribution. Nie et al. [12] presented an analytical solution for orthotropic FG beams with arbitrary graded material properties by the displacement function method. Thai and Vo [13] developed the FG beam for bending and free vibration analysis. According to this paper, the analytical solutions were presented by means of Hamilton's principle, and the developed theories accounted for higher-order variation of transverse shear strain through the depth of beam and satisfied stress-free boundary conditions on top and bottom surfaces. Furthermore, several influences of graduation laws, loading, boundary conditions and shear deformation on displacement, stress and frequency were investigated. Vo et al. [14] investigated the static behavior of FG sandwich beams by using the finite element model based on a quasi-3D theory. Various symmetric and non-symmetric sandwich beams with FGMs under the uniformly distributed load were considered. They showed that the effect of normal strain was important and should be considered in static behavior of sandwich beams. Recently, Filippi et al. [15] performed a static analysis of a functionally graded beam based on various theories and finite elements. By employing principle of virtual displacements in a weak form, the governing equations were derived considering different boundary and loading conditions, arbitrary material

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distributions and dimensions. Moreover, the stresses and displacements were compared with 1-, 2-, 3-D solutions.

However, the above-mentioned studies were restricted to the static analysis of FG beams with rectangular cross-sections. In general, thin-walled beams made of anisotropic materials show the complicated and coupled structural behavior. Thus, warping and other coupling effects can become of practical importance and should be considered in design process. A general thin-walled beam theory was well established by Vlasov [16] and later various thin-walled laminated composite beam theories were developed by many researchers [17–22]. With functionally graded materials, the gradual law in monotonous variation of volume fraction exhibits strong effects on behavior of beam.

Consequently, the objective of this paper is to concentrate on several aspects of materials distribution in study thin-walled FG beams, also present a general analytical model based on the variational formulation. The considered sections are mono-symmetric I- and channel-sections. Center of gravity and shear center of cross-sections are defined in a function of geometries and material properties. Governing equations are derived and solved such that the accurate deflection and twist angle can be obtained. In order to show the validity of this study, several numerical examples are presented. The important points of this study are summarized as follows

- The model is based on the Euler–Bernoulli type beam, Vlasov's thin-walled bar theory, and accounts for all structural coupling coming from the material anisotropy and warping of cross-sections.
- The general governing equations are derived by using a variational formulation.
- The locations of center of gravity and shear center are derived for FG beams with mono-symmetric I- and channel-sections.
- Analytical solutions are presented for flexural and torsional analyses of FG beams with cantilevered boundary condition.
- The effects of power law index and thickness ratios of ceramic on the locations of center of gravity, shear center, flexural and torsional behaviors of thin-walled FG beams are investigated.

2. Kinematics

In this study, two sets of coordinate systems which are mutually interrelated are required: an orthogonal Cartesian coordinate global system (x, y, z) and an orthogonal local coordinate system (n, s, z) for a plate segment of beam as shown in Fig. 1. The x- and y-axes lie in the plane of cross-section and the z-axis parallels to the longitudinal axis of beam. While the n-axis is normal to the mid-surface of plate segment and the s-axis is tangent to the mid-surface and is directed along the contour line of cross-section. The (x, y, z) and (n, s, z) coordinate systems are related through an angle

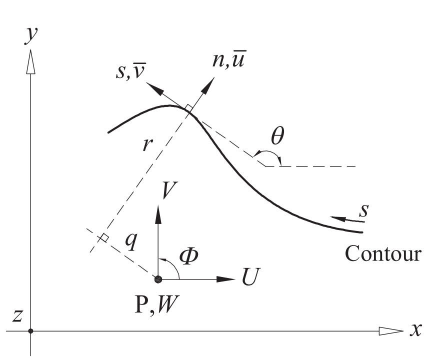

Fig. 1. Coordinates in thin-walled open cross-section.

of orientation θ as defined in Fig. 1. Point P is called the pole axis. To derive the analytical model for a thin-walled FG beam, the following assumptions are made

(i) The strains are assumed to be small.
(ii) The beam is linearly elastic and prismatic.
(iii) The contour of a cross-section does not deform in its own plane.
(iv) The shear strain $\bar{\gamma}_{sz}$ of the mid-surface is zero.

According to these assumptions, the displacements $\bar{u}$ and $\bar{v}$ of mid-surface in the contour coordinate system can be expressed in terms of displacements U and V at the pole P in x- and y-directions, respectively, and the rotation angle Φ, as follows

$$\bar{u}(s,z) = U(z)\sin\theta(s) - V(z)\cos\theta(s) - \Phi(z)q(s), \tag{1a}$$

$$\bar{v}(s,z) = U(z)\cos\theta(s) + V(z)\sin\theta(s) + \Phi(z)r(s). \tag{1b}$$

The out-of plane shell displacement $\bar{w}$ can now be found from the assumption iv. For each element of middle surface

$$\bar{\gamma}_{sz} = \frac{\partial\bar{v}}{\partial z} + \frac{\partial\bar{w}}{\partial s} = 0. \tag{2}$$

Substituting $\bar{v}$ in Eq. (1b) into Eq. (2) and integrating with respect to s from the origin to the arbitrary point on contour, the displacement $\bar{w}$ can be expressed as

$$\bar{w}(s,z) = W(z) - U'(z)x(s) - V'(z)y(s) - \Phi'(z)\omega(s), \tag{3}$$

where the superscript 'prime' denotes differentiation with respect to z and ω is the so-called sectorial coordinate or warping function given by

$$\omega(s) = \int_{s}^{s_0} r(s)\mathrm{d}s. \tag{4}$$

To derive Eq. (3), the following geometric relations

$$\cos\theta = \frac{\mathrm{d}x}{\mathrm{d}s}, \tag{5a}$$

$$\sin\theta = \frac{\mathrm{d}y}{\mathrm{d}s}, \tag{5b}$$

are used.

The displacement components u, v and w of any generic point on the profile section are given with respect to mid-surface displacements as

$$u(n,s,z) = \bar{u}(s,z), \tag{6a}$$

$$v(n,s,z) = \bar{v}(s,z) - n\frac{\partial\bar{u}(s,z)}{\partial s}, \tag{6b}$$

$$w(n,s,z) = \bar{w}(s,z) - n\frac{\partial\bar{u}(s,z)}{\partial z}. \tag{6c}$$

3. Strain energy

The linear strains with the small displacement are given by

$$\epsilon_z = \frac{\partial w}{\partial z} = \frac{\partial\bar{w}}{\partial z} - n\frac{\partial^2\bar{u}}{\partial z^2} = \bar{\epsilon}_z + n\bar{\kappa}_z, \tag{7a}$$

$$\gamma_{sz} = \bar{\gamma}_{sz} + n\bar{\kappa}_{sz}, \tag{7b}$$

where $\bar{\epsilon}_z$ and $\bar{\kappa}_z$ are axial strain and curvature of the shell, respectively, expressed as

$$\bar{\epsilon}_z = \epsilon_z^0 + x\kappa_y + y\kappa_x + \omega\kappa_\omega, \tag{8a}$$