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Post-buckling of sigmoid-functionally graded material toroidal shell segment surrounded by an elastic foundation under thermo-mechanical loads

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ABSTRACT

The nonlinear buckling and post-buckling of ceramic–metal–ceramic layers (S-FGM) toroidal shell segment surrounded by elastic foundation under thermo-mechanical loads are investigated with an analytical approach in this paper. Based on the classical thin shell theory with geometrical nonlinearity in von Karman–Donnell sense, Stein and McElman assumption and Pasternak foundation model, the governing equations of nonlinear buckling of S-FGM toroidal shell segment are analyzed. The static critical buckling loads and the post-buckling analyses in two cases – movable and immovable boundary conditions including temperature effects are obtained. Furthermore, the effects of geometry ratios, characteristic of materials, elastic foundation and thermal environment on the nonlinear buckling of shells are presented.

1. Introduction

Japanese scientists firstly founded the functionally graded materials in Sendai area in 1984 [1]. Ever since then, a myriad of studies on this material have been published, attracting scholarly attention worldwide. Functionally graded materials are composite materials composed of two phases: ceramic and metallic constituent materials. Mechanical and physical features of FGMs are better than fiber reinforced laminated composite materials because of such properties as stress concentration, oxidation resistance, high toughness, and heat-resistance. Hence, FGMs are used to manufacture heat-resistant and lightweight structures in the aerospace industry, mechanical and medical industry and so forth. Therefore, the nonlinear buckling and post-buckling problem of FGM structures have fueled a great deal of research.

Many FGM structures on elastic media have been studied for a long time by many scientists. The simplest model for the elastic foundation is Winkler [2], which features a series of separated springs without coupling effects between each other. Then the model was expanded by Pasternak [3] to incorporate a shear layer. Bagherizadeh et al. [4] investigated the mechanical buckling of functionally graded material cylindrical shells surrounded by Pasternak elastic foundation. Theoretical formulations were presented based on a higher-order shear deformation shell theory. Sofiyev [5–8] studied the buckling of FGM shells on elastic foundation. Moreover, the post-buckling of FGM cylindrical shells surrounded by an elastic medium was presented by Shen [9–11]. Bich et al. [12,13] gave an analytical approach to present the nonlinear vibration and buckling for FGM shell on the elastic foundation. The static, dynamic and vibration analyses of FGM doubly curved panel resting on Pasternak-type elastic foundation based on the first order shear deformation and the modified Sanders shell theories using the Navier type solution and the Laplace transform were investigated by Kiani et al. [14].

Noda [15] and Praveen et al. [16] first discovered the heat-resistant FGM structures and studied temperature-dependent material properties in thermo-elastic analyses. The postbuckling analysis of axially-loaded functionally graded cylindrical shells in thermal environments using the classical shell theory with von Kármán–Donnell-type of kinematic nonlinearity was investigated by Shen [17]. In Shen [18], the post-buckling analysis of imperfect stiffened laminated cylindrical shell under combined external pressure and thermal loading using the formulation based on a boundary layer theory of shell buckling including the effects of nonlinear prebuckling deformations, nonlinear large deflection in the postbuckling range and initial geometrical imperfections of

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the shell was studied. Kadoli and Ganesan [19] studied the linear thermal buckling and free vibration for functionally graded cylindrical shells subjected to a clamped–clamped boundary condition with temperature-dependent material properties. Furthermore, the analytical approach to investigate the nonlinear axisymmetric response of functionally graded shallow spherical shells subjected to uniform external pressure incorporating the effects of temperature was given by Bich and Tung [20]. Boroujerdy and Eslami [21] investigated the thermomechanical instability of FGM shallow spherical shells and surface-bonded piezoelectric actuators based on the classical shell theory and Sanders nonlinear kinematics equations. An analytic approach for thermoelastic bending of FGM cylindrical shell under a uniform transverse mechanical load and non-uniform thermal loads using the equations with the radial defection and horizontal displacement was discussed by Dai and Dai [22]. In addition, Eslami et al. [23] pointed out bifurcation behavior of heated FGM conical shell. The heat conduction equation of the shell was resolved based on an iterative generalized differential quadrature method. General nonlinear equilibrium equations and the associated boundary conditions were obtained using the virtual displacement principle in the Donnell sense. Huang and Han [24,25] investigated the nonlinear postbuckling behaviors of functionally graded cylindrical shells under uniform radial pressure using the nonlinear large deflection theory of cylindrical shells with the temperature-dependent material properties taken into account. In the analysis, the nonlinear strain–displacement relations of large deformation and the Ritz energy method were used while by taking the temperature-dependent material properties into account; various effects of the external thermal environment were also investigated. Dung and Hoa [26] studied the nonlinear buckling of eccentrically stiffened functionally graded circular cylindrical shells under external pressure, using approximate three-terms solution of deflection and Galerkin's method to give explicit expression for critical load and postbuckling load–deflection curves. Duc and Thang [27] researched the nonlinear response of imperfect eccentrically stiffened S-FGM thin circular cylindrical shells surrounded on elastic foundation under uniform radial load. The approximate solution of deflection in this paper, however, was only one-term with linear buckling shape and obtained postbuckling equilibrium paths incompletely illustrated the nonlinear response of the shell. The more correct on can be seen in [26]. Based on third order shear deformation shell theory, the buckling analysis of a two dimensional FGM cylindrical shell embedded in an outer elastic medium under combined axial and transverse loading was investigated by Allahkarami et al. [28]. Francesco et al. [29] analyzed recovery of through-the-thickness transverse normal and shear strains and stresses in statically deformed FG doubly-curved sandwich shell structures and shells of revolution using the generalized zigzag displacement field and the Carrera Unified Formulation. Three different through-the-thickness distributions of the volume fractions of constituents and two different homogenization techniques were employed to deduce the effective moduli of linear elastic isotropic materials.

Toroidal shell segment has been used in such applications as satellite support structures, fusion reactor vessels, rocket fuel tanks, diver's oxygen tanks and underwater toroidal pressure hull. Today, FGMs consisting of metal and ceramic constituents have received remarkable attention in structural applications. The smooth and continuous change in material properties enables FGMs to avoid interface problems and unexpected thermal stress concentrations. Some components of the above-mentioned structures may be made of FGM. Stein and McElman [30] investigated the homogenous and isotropic toroidal shell segments with the buckling problem. McElman [31] carried out the eccentrically stiffened shallow shells of double curvature with the static and dynamic behaviors in NASA technical note. The initial post-buckling behavior of toroidal shell segments subject to several loading conditions based on the basic of Koiter's general theory was studied by Hutchinson [32]. Recently, there have been some new publications about toroidal shell segment structures. Bich et al. [33] studied the buckling of eccentrically stiffened functionally graded toroidal shell segment under axial compression, lateral pressure and hydrostatic pressure based on the classical thin shell theory, the smeared stiffeners technique and the adjacent equilibrium criterion. Furthermore, the nonlinear buckling and post-buckling problems of ES-FGM surrounded by an elastic medium under torsional load based on the analytical approach are investigated by Bich et al. [34].

To the best of the authors' knowledge, this is the first time an analytical approach to the nonlinear buckling of sigmoid FGM toroidal shell segments subjected to lateral pressure surrounded by an elastic foundation and in a thermal environment has studied.

In the present paper, the nonlinear buckling and post-buckling behaviors of S-FGM toroidal shell segments surrounded by an elastic medium under lateral pressure loads including temperature effects are investigated. The governing equations are derived based on the classical shell theory with the nonlinear strain–displacement relation of large deflection. Moreover, the three-term solution of deflection including the linear buckling and nonlinear buckling shape is chosen. The Galerkin method is used for the nonlinear buckling analysis of shells to give closed-form expressions of the critical buckling load and the relation between deflection and lateral load. The specific features on the critical buckling loads and post-buckling curves for convex and concave shell in two cases – movable and immovable boundary conditions including temperature effects are investigated. Furthermore, the influence of mechanical properties of two material structures S-FGM and P-FGM can be analyzed. Effects of buckling modes, geometrical parameters, volume fraction index, elastic foundation and temperature on the nonlinear buckling behavior of shells are also considered.

2. Sigmoid functionally graded material (S-FGM)

Denote V_m and V_c as volume-fractions of metal and ceramic phases respectively, where $V_m + V_c = 1$ and

$$V_m \text{ is expressed as } V_m(z) = \begin{cases} \left(\frac{2z+h}{h}\right)^k, & k \geqslant 0,\ -\frac{h}{2} \leqslant z \leqslant 0 \\ \left(\frac{-2z+h}{h}\right)^k, & 0 \leqslant z \leqslant \frac{h}{2} \end{cases}, \quad (1)$$

where h is the thickness of thin – walled structure, k is the volume – fraction exponent ($k \geqslant 0$); z is the thickness coordinate and varies from $-h/2$ to $h/2$; the subscripts m and c refer to the metal and ceramic constituents respectively. Fig. 1 describes the material characteristic of sigmoid FGM. According to the mentioned law, the Young modulus $E(z)$ and the thermal expansion coefficient $\alpha(z)$ can be expressed in the form

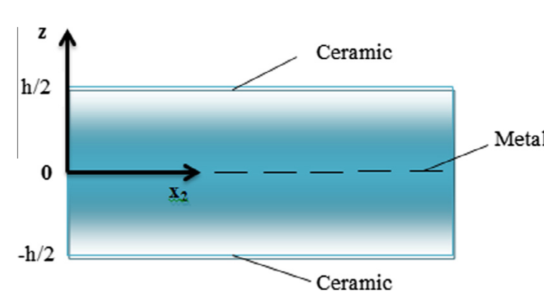

Fig. 1. The material characteristic of sigmoid functionally graded material.