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Multi-objective robust optimisation of unidirectional carbon/glass fibre reinforced hybrid composites under flexural loading

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ABSTRACT

A multi-objective robust optimisation (MORO) of carbon and glass fibre-reinforced hybrid composites under flexural loading based on an *a posteriori* approach has been presented in this paper. The hybrid composite comprised of T700S carbon/epoxy laminate at the tensile side and E glass/epoxy laminate at the compressive side. The conflicting objectives for optimisation were to minimise the cost and weight of the composite subject to the constraint of a minimum specified flexural strength. Fibre angles and thicknesses of each lamina were considered as uncertain but bounded variables with the worst-case analyses being performed as a non-probabilistic method and the effect of uncertainties being determined. A hybrid multi-objective optimisation evolutionary algorithm (MOEA) was introduced through modification of an elitist non-dominated sorting genetic algorithm (NSGA-II) and combining it with the fractional factorial design method. The performance of the hybrid algorithm was found to be superior to that of the original version of NSGA-II. The multi-objective robust optimisation of the hybrid composite was solved by utilising the proposed algorithm for several levels of strength with the robust Pareto optimal sets being generated and compared. Three scenarios have been considered to illustrate the applicability of the obtained solutions in an *a posteriori* decision making process.

1. Introduction

Hybrid composite laminates, which contain more than one type of reinforcement phase, *e.g.*, carbon and glass fibre, are widely used in applications requiring high strength, low weight and low cost. For example, since glass fibres possess a lower modulus and higher strain-to-failure when compared to carbon fibres, the flexural strength of carbon fibre-reinforced polymer (CFRP) composites can be significantly improved by replacing some of the carbon fibre laminas at the compressive side of the composite by glass fibre laminas [1–6].

However, the effect of hybridization in fibre-reinforced composites is not always positive [7] and finding the optimum level of hybridization would be of fundamental concern for the design of such materials. The hybrid effect is defined as the deviation of a certain property from the rule of mixtures (RoM) equation. The general rule of mixtures is the weighted mean of a material property with respect to the volume fraction of the constituents. Dong et al. [3,4,8] investigated the optimal design of carbon and glass fibre-reinforced hybrid composites under bending load and concluded that the fibre volume fraction of the glass/epoxy laminas must be higher than that of the carbon/epoxy laminas in order to achieve a positive hybrid effect on flexural strength. They also determined the critical level of hybridization (the critical hybrid ratio) in which the hybrid effect is maximised.

The hybridization of carbon and glass fibre influences not only the flexural strength and stiffness but other properties such as cost and weight. Since the glass fibres are heavier and cheaper than carbon fibres, hybridization of a CFRP composite through the incorporation of glass fibres leads to a lower material cost but higher density. Minimisation of weight and cost as two conflicting objectives is a continuing concern in the design process of hybrid composite structures. Several studies have attempted to define and solve the multi-objective optimisation of hybrid composites with regards to minimum cost and/or weight [9–16].

An optimisation problem involving more than one simultaneous objective function is referred to as multi-objective optimisation with the solution for such a problem being achieved through the trade-off between objectives. A set of trade-off solutions that cannot be improved with respect to one objective without hurting another objective is known as a Pareto set and referred to as a Pareto optimal front when plotted in the design space [17]. The multi-objective optimisers aim to find the optimal solutions which

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form the ideal Pareto set with respect to all objectives. The majority of classical multi-objective optimisation methods avoid the complexities of multiple objectives and simply transform the problem into a single objective using *a priori* methods and preference-based strategies [18]. However, choosing a reliable and accurate preference of the objectives requires higher-level information which may not be available during the initial stages of the design process [17]. Unlike *a priori* methods, *a posteriori* methods aim at generating the Pareto optimal sets regardless of the objective preferences. Thus, sets of trade-off solutions can be generated which allow additional evaluation and comparison by the designer in order to make a final decision. To this end, a multi-objective optimisation evolutionary algorithm (MOEA), which is classified as an *a posteriori* method, can be utilised to produce a set of Pareto optimal sets in a single simulation and hence improve the solutions in a number of evolutions without considering any preference of objectives. There are a number of MOEAs available, *e.g.*, strength Pareto evolutionary algorithm (SPEA-II) [19], Pareto archived evolutionary strategies (PAES) [20] and non-dominated sorting genetic algorithm (NSGA-II) [21], which have been applied in order to generate optimal Pareto sets. Amongst these, NSGA-II is the most popular due to its effectiveness and simplicity with this algorithm being based on non-domination and crowding distance sorting and generates populations through the use of genetic operators, thus leading to optimal solutions, in a number of iterations known as a generation. However, there is no guarantee that population members progress in all iterations. To overcome the convergence problem and make the overall procedure faster the NSGA-II process should be combined with one or more mathematical optimisation methods having local convergence properties [17,22].

Most of the research in the field of multi-objective optimisation of composites has used preference-based classical methods to convert the multi-objective optimisation problem into a single objective form [15,16,23–25,11,26,27]. For example, Hemmatian et al. [15,16] applied a gravitational search algorithm and elitist ant system to solve the multi-objective optimisation of carbon/glass fibre hybrid composites in order to achieve designs with minimum weight and cost. They considered first natural frequency as a constraint and simply used the weighted sum method (WSM) to construct Pareto-optimal fronts. In contrast to this, Walker et al. [11] used a sequential optimisation procedure to minimise the weight and cost of symmetric carbon/glass/Kevlar hybrid laminated plates subject to a buckling load. Relatively little research has been conducted in this area using evolutionary methods such as Lakshmi and Rao [28] who used a new hybridized version of NSGA-II to minimise the weight and cost of laminated hybrid composite cylindrical shells with the hybrid algorithm being superior in performance. Visweswaraiah et al. [29] used both *a priori* and *a posteriori* methods to optimise a composite helicopter blade and demonstrated the trade-off designs given by their evolutionary non-dominated sorting hybrid algorithm (NSHA) which could not be achieved by *a priori* classical methods. Recently, Madeira et al. [30] used the Direct MultiSearch (DMS) method based on a derivative free solver for multi-objective optimisation problems to find the optimal design of viscoelastic laminated sandwich composite panels for maximum modal damping, minimum mass and material cost.

Traditionally, studies in the field of optimisation of composites have assumed deterministic values for design variables and ignored uncertainties in material properties and geometry tolerances. However, practical engineering design optimisation problems in the field of composite materials contain variations in design parameters due to reasons such as manufacturing tolerances, defects, voids, fibre misalignment and the presence of resin rich regions [31–34]. Such variations in manufacturing and material properties will generally degrade the performance of the optimal design. Thus, the mechanical properties of highly optimised composites designed with the assumption of deterministic parameters may be significantly lower in practice and fail to satisfy the required performance constraints. Robust design philosophy, which was introduced by Taguchi [35], aims to obtain optimal solutions that are insensitive to such material and manufacturing variations. Methods for the modelling of material uncertainties may be classified into probabilistic and non-probabilistic. For the case of probabilistic methods, a probability distribution for the uncertain parameters is assumed with the variation of the objective(s) being minimised and the mean value of the objective(s) being optimised. However, accurate estimates of probability distributions in the early stages of the design process may be difficult or sometimes impossible to achieve [36]. On the other hand, non-probabilistic methods do not require the estimation of any probability distributions but instead utilise information concerning the bounds of any uncertain parameters to ensure the suitability of optimum solutions when the uncertainties are taken into account [37]. Several studies have attempted to investigate the robust optimisation of composites [38–44] such as Radebe and Adali [38] who studied hybrid cross-ply cylinders subject to external pressure and considered uncertainties in the elastic constants of the materials. They determined the worst-case combination of material uncertainties to find the minimum buckling load. The elastic constants were considered as uncertain-but-bounded variables with the minimum cost designs being found. Recently, the current authors proposed a robustness index for the consideration of uncertainties in lamina thickness in optimisation problems [44]. However, the multi-objective optimisation problem was converted to a single objective form by using the WSM whilst uncertainties in fibre angles were not included.

In the present study, a hybrid multi-optimisation evolutionary algorithm was proposed by modifying and combining the NSGA-II process with a local search which uses the fractional factorial design method in order to improve the convergence rate. The original NSGA-II and modified algorithms were employed to find the robust Pareto optimal sets of carbon and glass fibre-reinforced epoxy hybrid composite plates under bending load and their performance was evaluated and compared. The conflicting objectives of the optimisation were to minimise the weight and cost subject to the constraint that the flexural strength be greater than a specified value. The fibre volume fraction and thickness of the carbon and glass epoxy lamina were considered as the design variables with the thickness of the lamina and fibre angle being considered to be uncertain-but-bounded variables. Uncertainties were modelled using the worst-case analysis as a non-probabilistic method. The optimisation problem was formulated and solved for several flexural strengths with the Pareto optimal sets being determined. Three example scenarios were considered to show the applicability of the solutions in *a posteriori* methods.

2. Hybrid composite model

In this study, a unidirectional carbon and glass fibre-reinforced epoxy laminate hybrid composite under three-point bending was investigated. Recent work by the current authors [1,3,4] has indicated that a positive hybrid effect for the flexural strength can be achieved by replacing a portion of the carbon fibres by glass fibres at the compressive side of the specimen. Therefore, the same stacking configuration was used in the present work as shown in Fig. 1 with the properties of the fibres and matrix being presented in Table 1 The span-to-depth ratio, L/h, was relatively large so that the plate can be considered as a thin plate. Roller supports were used at both ends with a load, F, being applied at the mid-span. The thicknesses of the carbon/epoxy, h_c, and glass/epoxy, h_g, sections, in addition to the fibre orientations, were considered as uncertain variables that varied about their nominal value.