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# Thermal stresses in hybrid materials with auxetic inclusions

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ABSTRACT

We study thermo-elastic behaviour of the hybrid materials with auxetic inclusions. To this end we analyse representative volume elements consisting of a large number of randomly positioned cubic inclusions using the finite element method and determine the effective coefficient of thermal expansion and the average thermal stresses in constrained hybrid materials developed as a result of a uniform temperature change. We study the effect of Poisson's ratio, Young's modulus and the coefficient of thermal expansion of the inclusions on the effective coefficient of thermal expansion and thermal stresses. We demonstrate that the auxeticity of the inclusions can reduce thermal stresses, which is controlled by the values of the Poisson's ratio and the coefficients of thermal expansion of the inclusions and matrix.



## 1. Introduction

Particle reinforced composites are a common form of hybrid materials used in engineering structures. The particles are added to a matrix to alter the overall properties of the hybrid in terms of mechanical moduli, strength, electric and thermal conductivity and etc. [1] and thus improve performance and enhance functionality. In general, these effective properties depend upon the properties of both matrix and inclusions, concentration of the inclusions and their shape. Recently we reported on a new class of hybrid materials consisting of auxetic (possessing negative Poisson's ratio (NPR)) and non-auxetic phases [2]. We demonstrated that by adding auxetic inclusions in a non-auxetic matrix one could achieve a considerable increase in stiffness over the stiffness of the matrix and inclusions. We studied the influence of various parameters on the mechanical performance of these hybrids. We showed that the stiffening effect is controlled by the values of the Poisson's ratio of the matrix and inclusions, shape of the auxetic inclusions and their concentration [2]. In the present study, we consider the thermal deformation of the hybrid materials with auxetic inclusions. We evaluate the coefficient of thermal expansion (CTE) and thermal stresses occurring in these hybrids.

Thermal deformations and thermally induced stresses play a critical role in many engineering applications ranging from microelectronic and optic devices [3] to gas turbines and jet engines [4,5]. The thermal stresses are a common source of mechanical failures, such as brittle fracture, excessive deformations, thermal fatigue, material degradation and buckling. In addition, the thermal distortions cause functional failures in systems that are sensitive to large deformations, such as high precision optics and electronics [3]. One of the ways to manage the thermo-mechanical design is to employ materials with enhanced thermal properties [4] or in multipart cases with matching thermal properties [6]. In this sense, the hybrid material approach as introduced by Ashby and Bréchet [1] offers a possibility to create the materials optimally tuned to the purpose. The most important feature of this concept is that the internal architecture and scale are introduced as design parameters. In particular, the separation of scales allows the design of components with unusual properties, where the effective properties of the components are first defined by averaging over suitable representative volume elements at the component scale and then the effective characteristics of the hybrid are designed at the upper scale based on these properties (e.g. [2]).

Following this approach the control of the thermal properties of a multiphase composite can be carried out in two ways: (i) by designing the microstructure of the hybrid that would exhibit the desired thermal expansion behaviour at the macroscale level or (ii) by altering the effective thermal and mechanical properties of the phases. The first approach has been widely used to design hybrid microstructures leading to the negative thermal expansion (NTE) using periodical lattices, cellular structures and kinematic actuators. The NTE effect in these microstructures was achieved through various bimaterial elements at the microscale (the same scale as the microstructure). One of the popular designs was a lattice consisting of bimaterial cells composed of a lower CTE stiff

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frame and a higher CTE soft core [7–12]. A range of the lattices with low or negative CTE that was developed capitalising on this concept includes rigidly connected curved hexagonal honeycombs [8], rigidly connected cells with straight ribs [9,10,12], non-identical cells [11] and thin films [12]. In models of the cellular microstructures with tunable CTE, the biomaterial approach was applied to design the cell ribs. This included the cells with curved bimaterial ribs [13], and 2D and 3D lattices composed of nested biomaterial tubes [14,15]. The periodic frame structures consisting of rods with altering individual CTEs were also shown to exhibit the NTE effect [16,17]. A systematic approach to design of NTE microstructures based on topology optimisation of the microstructural architecture using a library of shapes so-called Freedom, Actuation, and Constraint Topologies (FACT) was explored in [18].

The second approach to the design of multiphase composites with tunable thermal properties capitalises on the concept of particle-reinforced composites. In this case, the control over the thermal performance of the hybrid is achieved by introducing the inclusions of different shapes with special thermal and mechanical properties [19,20]. The discovery of tungstate and molybdate compounds [21,22] exhibiting isotropic NTE inspired the use of these materials for improving the thermal performance of composites [19,20,23,24]. At the same time, the use of other materials was also investigated including negative stiffness phase [25] and negative Poisson's ratio [26,27]. The shapes and arrangement of the phases were also considered. Hard disc, cylindrical and needle shapes inclusions with high positive CTE embedded into a soft matrix with low positive CTE were shown exhibiting the NTE under certain conditions [28]. Laminated stacks with alternating auxetic and non-auxetic layers with negative and positive CTE can be designed to provide low or even negative CTE [29].

Here we consider the application of auxetic inclusions with negative and positive CTE. The auxetic materials have attracted vast research attention during last two decades. A range of auxetic systems that have been identified and studied included polymers and foams (e.g. [30–33]), molecular models [34–38], auxetic crystals (e.g. [39–41]), nanoauxetics [42–44], composites (e.g. [45–51]), auxetic lattices [52,53], hollow sphere stacks [54] and auxetic metamaterials (e.g. [55–59]). The studies on the auxetic behaviour demonstrated many beneficial properties associated with the auxetic behaviour, such as improved shear modulus, enhanced fatigue, indentation and impact resistance [60–62]. It was also shown that in some cases the auxetic behaviour reduces the stress created as a result of the thermal expansion [27]. The main objective of this paper is to study the influence of the auxetic phase on the thermal stresses in hybrid materials.

This paper is structured as follows. First, we introduce a representative volume element with randomly distributed cubic auxetic inclusions [2,63] and analyse it using the finite element method. Then we compare the obtained effective CTEs with the existing methods for evaluation of effective characteristics of two-phase composites. Finally, we investigate the thermal stress levels in the hybrids with auxetic inclusions with positive and negative CTE and various Young's moduli and negative Poisson's ratio.

## 2. Thermo-elastic behaviour of hybrid materials with auxetic inclusions. Finite element model

Consider a hybrid consisting of an isotropic matrix with conventional material properties (positive Poisson's ratio and positive thermal expansion coefficient) and auxetic inclusions. We assume that the auxetic inclusions have cubic shape and may exhibit both positive and negative thermal expansion. We also assume that the separation of scales exists implying that one can introduce a volume element (a representative volume element) with the dimensions that are much larger than the sizes of inclusions and distances between them, while still much smaller than the characteristic length of variations of the externally applied fields. Then the effective characteristics of the hybrid (e.g. effective moduli, effective coefficient of thermal expansion) can be calculated through the fields averaged over the volume element. To evaluate the continuum properties of the hybrid we model a representative volume element (RVE) of a volume $V$ by the finite element method as shown in Fig. 1. The RVE is divided into a regular cubic grid, in which each cube (depicted by a solid line in Fig. 1) represents either the inclusion or a part of the matrix. Each inclusion is meshed using 3D finite elements that are also regularly arranged into the cubic grid plotted by dash lines in Fig. 1. It is also assumed that the bond between the inclusions and the matrix is rigid. The auxetic inclusions are randomly distributed and allowed to form clusters of complex geometry typical at high volumetric fractions of the inclusions. A particular computational realisation of the hybrid is achieved by randomly assigning the material properties to the cubic elements in the grid. As all blocks have the same dimensions, the concentration of auxetic inclusions is defined as the ratio of the number of the blocks assigned the negative Poisson's ratio to the total number of the blocks. The negative and positive Poisson's ratio blocks are shuffled using the random permutation function incorporated in MATLAB. This function uses the Mersenne-Twister pseudo-random number generator producing a uniform distribution over the total number of blocks [64].

We study the thermo-elastic behaviour of the hybrid by considering two cases: the free thermal expansion test for evaluation of the effective CTE and the confined thermal loading test for the evaluation of the thermal stresses. The computational models for the simulation of these conditions are outlined next.

### 2.1. Evaluation of the effective coefficient of thermal expansion

To determine the effective CTE we apply a uniform change of temperature $\Delta T$ to the unconstrained volume of the hybrid. We assume that the RVE is symmetric in all three directions and analyse only an eighth of the cubic RVE, Fig. 2. Thus, the following symmetry conditions are imposed at the planes of symmetry:

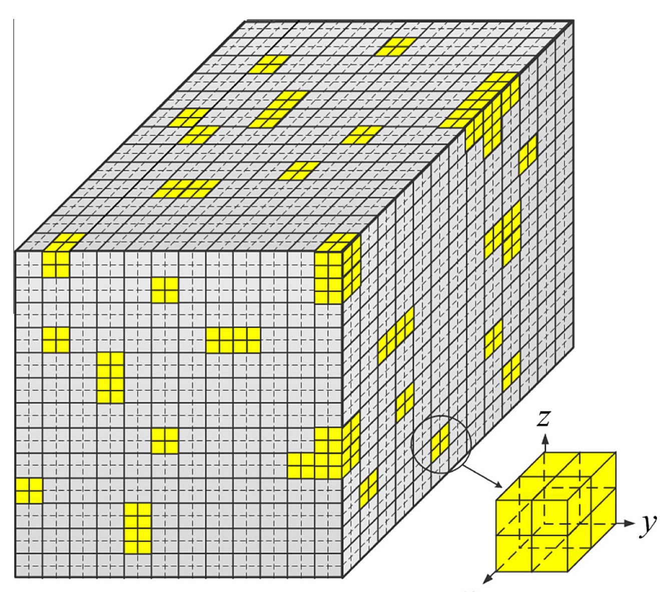


**Fig. 1.** Representative volume element for the hybrid material with randomly distributed auxetic cubic inclusions. Each cubic inclusion is modelled by a number of 3D finite elements.