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Comparative study on thin and thick walled cylinder models subjected to thermo-mechanical loading

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ABSTRACT

This paper gives a theoretical background and compares two analytical approaches, thin- and thick-walled models, analyzing composite cylindrical tubes under thermo-mechanical loadings. First, a theoretical background is introduced, and a lamination theory and an elasticity theory for thick-wall tubes are recalled. A systematic parametric study for various geometrical, material and load settings was performed to find out the difference between analyzed calculation approaches. It was generally observed, that the Classical Lamination Theory can be successfully applied for pressure loads, however this plane-stress assumption may generate remarkable errors if thermal loads are introduced. It is especially the case for highly orthotropic cylinders. The generalization of the achieved results allowed to recommend a new criterion for the selection of an appropriate calculation model. The proposed measure incorporates simple forms of tubes' geometrical parameters (D/t) and material factor (C_{22}/C_{33}). Thanks to the applied approach the importance of through-thickness stresses can be quickly assessed.

1. Introduction

The mechanical behavior of pressurized cylinders made of an isotropic material is very well elaborated. The literature delivers a thick walled model based on Lamé theory, which incorporates principle stresses in all three directions, and a thin wall model, which neglects radial stresses [1–7]. Normally, it is assumed that if tube's diameter-to-thickness ratio, D/t is more than 20, the radial stresses are an order of magnitude smaller than the other stress components. In this case a simplified, thin-wall assumption could be successfully applied. This approach may not work however, if a laminated cylinder is considered. In this case stresses within the tube are not related only to D/t ratio, but may be dependent on material properties (different for various plies), and lay-up design (layer thicknesses and orientations). For this reason it is frequently accepted that diameter-to-thickness ratio cannot be treated as the only factor allowing to decide if a plane-stress model may be used for a particular design case.

The fundamental theoretical background for analysis of the anisotropic bodies was provided by Lakhnitskii [8], and his work has been referenced later in the large number of textbooks dealing with composites [9–12]. The application of the orthotropic material model into cylindrical structures was given by Scherrer [13], Pagano [14], Wilson and Orgill [15], and Pindera [16]. The solid description of the analytical solution for the laminated circular tube subjected to the mechanical loading was provided by Herakovitch [17]. Also hybrid structures, like Fiber Reinforced Metal (FRM), or Fiber Metal Laminates (FML) are of interest, [18–20]. Today, with growing popularity of numerical methods, theoretical investigations are strongly supported by FEM analyses [21–23].

However, even if the literature covering the theory and practice of composite cylinders is quite reach, the systematic studies comparing two different calculation models, thick- and thin-walled, are not common. The work described in this paper compares both above approaches. The basic information about a classical lamination concept and an elasticity theory for thick-walled orthotropic tubes is given in Section 2, providing the insight into the applied material models and constitutive relations. Next, the numerical example is presented, and calculations managed for different loads, diameter-to-thickness ratios and material properties are described. The outcome of the study delivers a proposal of new criterion allowing to assess if the plane-stress assumption could be safely applied to the particular design case. The concluding remarks are given in the last section of the article.

2. Theoretical investigation

2.1. Isotropic cylinder

In order to investigate a difference between thin- and thick walled theories for pressurized cylinders, the analysis of an

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isotropic structure will be recalled. The well known Lamé theory states that the hoop and radial stresses in a pressurized cylinder can be described as:

$$\sigma_{\Phi,r} = p\frac{R_1^2}{R_2^2 - R_1^2}\left(1 \pm \frac{R_2^2}{r^2}\right) \quad (1)$$

where p in an internal pressure acting on the inner surface, and R_1 and R_2 are the inner and outer radii, respectively.

Focusing only on the highly stressed internal surface ($r = R_1$), and expressing the diameter-to-thickness ratio as K, one may reformulate Eq. (1) to:

$$\sigma_{\Phi,r} = p\left[\frac{K^2}{4(K+1)} \pm \frac{K^2}{4(K+1)} \pm 1\right] \quad (2)$$

where:

$$K = \frac{D_1}{t} = \frac{2R_1}{R_2 - R_1} \quad (3)$$

where D_1 is an internal diameter and t is the wall thickness of the cylinder.

Based on Eq. (2) it is possible to state, that for large diameter-to-thickness ratio ($K \gg 10$) the hoop stress could be quite well estimated by Eq. (4), as proposed also by a thin-wall model:

$$\sigma_{\Phi} = p\left[\frac{K^2}{2(K+1)} + 1\right] \rightarrow p\frac{K}{2} = \frac{pD_1}{2t} = \tilde{\sigma}_{\Phi} \quad (4)$$

The relative error between Lamé approach and thin-wall model in the case of an isotropic, pressurized cylinder depends only on the diameter-to-thickness ratio, and may be calculated as:

$$\delta = \frac{\sigma_{\Phi} - \tilde{\sigma}_{\Phi}}{\sigma_{\Phi}} = \frac{K+2}{K^2 + 2K + 2} \approx \frac{1}{K}[\times 100\%] \quad (5)$$

With the help of Eq. (5) one can simply estimate that the application of the thin-wall model into a pressurized cylinder, having $K = 20$, can introduce an error in stress calculation at the level of about 5%. It should be also noted, that even smaller error may be achieved, if a mean diameter (not D_1) is used in Eq. (4).

Lamé theory states also, that radial stresses vary across the wall thickness – from the value equal to $-p$, at the inner surface, to zero – at the outer surface. While the thin-wall theory totally neglects stresses across the thickness direction. If an unconstraint isotropic cylinder subjected to the temperature load is considered, both models similarly predict that the thermal strains will not generate stresses. It is not the case for a laminated tube, where the thermal stresses will be generated.

However, the comparison between these two theories is not so straightforward, if an anisotropic material, or orthotropic composite should be investigated. In this case the difference is not only affected by geometrical parameters of the cylinder, but also the material properties, which are different in principle directions.

2.2. Classical Lamination Theory

The thin-wall composite tube may be analyzed with the use of well-established Classical Lamination Theory, CLT. It basically considers plane-stress state, assuming that radial stresses in thin cylinders are significantly smaller than the other stress components, thus may be ignored. Such assumption simplifies the calculation process, therefore it is popular in industrial applications. In this classical approach [17] the stress–strain relation is characterized by an equivalent generalized force (N, M) – generalized strain (ε^0,κ) system:

$$\begin{Bmatrix} N \\ M \end{Bmatrix} + \begin{Bmatrix} N^T \\ M^T \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix} \quad (6)$$

where

N, M are vectors of forces {N_x, N_{Φ}, $N_{x\Phi}$} and moments {M_x, M_{Φ}, $M_{x\Phi}$}, respectively;

N^T, M^T refer to vectors of thermal forces and thermal moments, respectively;

ε^0, κ are vectors of strains due to in-plane forces and strains due to moments (curvatures), respectively;

A, D and B are called tension stiffness, bending stiffness and coupling stiffness matrices respectively. They are calculated as follows:

$$[A, B, D] = \int_{-t/2}^{t/2} [\bar{Q}]^k (1, z, z^2) dz \quad (7)$$

where $[Q]^k$ is the reduced stiffness matrix of a single kth lamina, which is spaced from the neutral plane of the laminate by distance z, and t is the total thickness of the laminate.

The reduced stiffness matrix $[Q]^k$ defines the relation between stresses and strains for a single lamina, as:

$$\begin{bmatrix} \sigma_x \\ \sigma_{\Phi} \\ \tau_{x\Phi} \end{bmatrix}^k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & 0 \\ 0 & 0 & \bar{Q}_{66} \end{bmatrix}^k \begin{bmatrix} \varepsilon_x \\ \varepsilon_{\Phi} \\ \gamma_{x\Phi} \end{bmatrix}^k \quad (8)$$

The vector of strains in a single lamina is an algebraic sum of mid-plane strains, curvatures and thermal strains:

$$\varepsilon^k = \varepsilon^0 + z\kappa - \varepsilon^T \quad (9)$$

where thermal strain vector ε^T reads:

$$\begin{bmatrix} \varepsilon_x^T \\ \varepsilon_{\Phi}^T \\ \gamma_{x\Phi}^T \end{bmatrix}^k = \Delta T \begin{bmatrix} \alpha_x \\ \alpha_{\Phi} \\ \alpha_{x\Phi} \end{bmatrix}^k \quad (10)$$

ΔT is the temperature change, and α_j are thermal expansion coefficients in respective directions.

It should be underlined, that the matrix B plays an important role in the lamination theory, since it causes complex interaction between the in-plane loads and bending effects (out-of-plane strains). However, composite structures are typically designed in such a way that all components of B matrix are zero, therefore generated stresses are only the result of in-plane strains, ε, driven by in-plane forces, N.

2.3. Thick-walled composite cylinder

The thick-walled analytical model used to study a composite cylinder under thermo-mechanical load assumes a general orthotropic laminate. In the most universal case, there are 15 unknowns (3 displacements, 6 strains and 6 stresses) to be derived from the equilibrium, constitutive, and continuity equations. Using these relations, and neglecting the radial shears, one can prove [17,23], that the radial, axial and tangential displacements can be calculated as:

$$\begin{aligned} w(r) &= Ar^{\lambda} + Br^{-\lambda} + \Gamma\varepsilon_x^0 r + \Omega\gamma^0 r^2 + \Psi r\Delta T \\ u(x,r) &= x\varepsilon_x^0 \\ v(x,r) &= xr\gamma^0 \end{aligned} \quad (11)$$

where: A, B, $\varepsilon_x{}^0$, γ^0 are constants of integration – to be determined from the boundary conditions, and: λ, Γ, Ω, Ψ are material coefficients, defined as:

$$\lambda = \sqrt{\frac{\overline{C}_{22}}{\overline{C}_{33}}}; \quad \Gamma = \frac{\overline{C}_{12} - \overline{C}_{23}}{\overline{C}_{33} - \overline{C}_{22}}; \quad \Omega = \frac{\overline{C}_{26} - 2\overline{C}_{36}}{4\overline{C}_{33} - \overline{C}_{22}};$$