Composite Structures 134 (2015) 147–157

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Analytical solution for bending and buckling analysis of functionally graded plates using inverse trigonometric shear deformation theory

Kamlesh Kulkarni, B.N. Singh *, D.K. Maiti [1]

Department of Aerospace Engineering, Indian Institute of Technology Kharagpur, Kharagpur, India

ARTICLE INFO

Article history:
Available online 22 August 2015

Keywords:
FGM
FGP
ITSDT
Navier solution
NPSDT

ABSTRACT

Functionally graded materials have become more popular in recent decades due to its ability of efficient utilization of the constituents materials. The structural functionally graded plate (FGP) has variation of the properties in the thickness direction according to power law or exponential law. A recently developed non-polynomial shear deformation theory named as inverse trigonometric shear deformation theory (ITSDT) has proved its accuracy and efficiency in modeling and analyses of laminated composite and sandwich structures. However, its efficiency for the FGP has not examined so far in the literature. In the present study, an attempt is made to extend ITSDT for the static and buckling analysis of FGP. An analytical solution for all edges simply supported FGP is proposed in this work. The bending analysis includes calculation of in-plane and transverse displacements, along with the calculation of in-plane and transverse normal and shear stresses. The buckling analysis includes calculation of critical buckling load for various conditions. Also, the effect of power index, aspect ratio, span to thickness ratio, uniaxial and biaxial loading are studied. From the results, it is observed that the theory accurately predicts the static and buckling responses of FGP.

1. Introduction

The functionally graded material (FGM), as the name suggests, is a composite material for which the properties of the material vary continuously in one or many directions. For the structural applications generally, functionally graded plates (FGPs) are used for which the properties vary along the thickness. The continuous variation of the properties, which may be according to power law or exponential law, ensures the smooth variation of transverse shear strength across the thickness. The FGPs find their applications in many advanced engineering industries such as aerospace, nuclear and biomedical. This increase in applicability of FGMs has attracted the attention of many researchers. Several research papers are available for the structural analyses of FGPs using elasticity solution as well as various shear deformation theories. The exact 3D solution by Kashtalyan [1] and Zenkour [2] provides benchmark for the analysis. Finding exact solution is a complex process and moreover it is possible only for a few special cases. Hence there is need of shear deformation theories which facilitates to find solution in much simpler way and with reasonable accuracy. Most of the shear deformation theories are two dimensional and assume plane stress condition which neglects transverse normal stress. The most fundamental deformation theory is classical plate theory (CPT) which assumes the plane normal to the mid-plane before bending remains plane and normal after bending. It neglects the effect of all transverse stresses and is less accurate. Hence it yields accurate results for thin plates only. Feldman and Aboudi [3], Javaheri and Islami [4] and Chi and Chung [5] have obtained static response of FGPs using CPT. The disadvantage of CPT was overcome by first order shear deformation theory (FSDT) proposed by Reissner [6] and Mindlin [7] which considers the effect of transverse shear deformation. According to FSDT, the transverse strains are constant throughout the thickness of plate which is unrealistic and requires a shear correction factor to nullify transverse shear strains on the top and bottom of the plate. Praveen and Reddy [8], Chinosi and Croce [9], Singha et al. [10], Alieldin et al. [11], Wen and Aliabadi [12] and Castellazzi et al. [13] adopted FSDT for the static analysis of FGPs. Nguyen et al. [14] also used FSDT but with modified shear correction factor, they showed that the shear correction factor for FGPs depends upon the power index. The third order shear deformation theory (TSDT) by Reddy [15] which is free from any shear correction factor and satisfies the condition of zero transverse shear strain at the top

* Corresponding author. Tel.: +91 3222 283026; fax: +91 3222 282242.
E-mail addresses: kam4aero@gmail.com (K. Kulkarni), bnsingh@aero.iitkgp.ernet.in (B.N. Singh), dkmaiti@aero.iitkgp.ernet.in (D.K. Maiti).
[1] Tel.: +91 3222 283028.

http://dx.doi.org/10.1016/j.compstruct.2015.08.060
0263-8223/

and bottom of the plate was employed by Reddy [16], Ferreira et al. [17], Oktem et al. [18], Tran et al. [19] and Taj et al. [20]. The third order shear deformation theory provided better results compared to CPT and FSDT but researchers have obtained more accurate results by adopting various non-polynomial shear deformation theories. In non-polynomial shear deformation theories, the in-plane displacements are the function of thickness coordinate. The function may be trigonometric, exponential or hyperbolic. Touretier [21] recommended sinusoidal function, Zenkour [22] suggested four variable theory with same function. Thai and Vo [23] also recommended sinusoidal function but they considered transverse deflection due to bending as well as due to shear. The tangential function was suggested by Mantari et al. [24]. Soldatos [25], Akvasi [26] and Mahi et al. [27] have suggested hyperbolic shear strain function for the analysis. Saldatos [25] used sine hyperbolic function; whereas, Akvasi [26] and Mahi et al. [27] suggested tangential hyperbolic shear strain function. Inverse trigonometric function is suggested by Thai et al. [28,29]. Thai et al. [28] used inverse tangential function; whereas, Thai et al. [29] proposed inverse tangential as well as inverse sinusoidal function. Exponential form of displacement field has been used by Karama et al. [30] and Mantari et al. [31]; whereas, Aydogdu [32] used logarithmic form of displacement field. Mantari et al. [33,34] have adopted the combination of trigonometric and exponential function for the analysis. Grover et al. [35] and Nguyen et al. [36] proposed a theory with inverse hyperbolic function. Grover et al. [35] proposed the theory for laminated plates and sandwich structures; whereas, Nguyen et al. [36] implemented the proposed theory for FGPs. The sinusoidal inverse hyperbolic function proposed by Grover et al. [35] has been adopted recently by Nguyen et al. [36] for the analysis of different FGPs showing that the function yields accurate results.

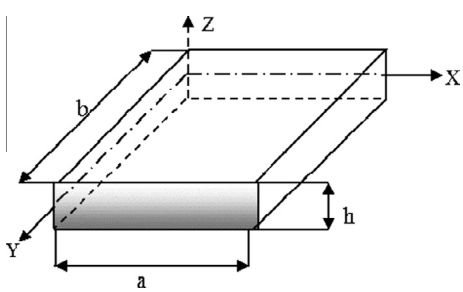

Fig. 1. Geometry of functionally graded plate.

In the present work, the shear deformation theory with cotangential inverse trigonometric function, developed by Grover et al. [38] has been extended for the bending and stability analyses of FGPs. The theory yields quite accurate results when applied to laminate and sandwich structures; however, the accuracy of prediction for FGPs is the aim of the present research work.

2. Mathematical formulation

The geometry of a functionally graded plate is as shown in Fig. 1. The dimensions of the plates are $a \times b \times h$, where 'a' is the length, 'b' is width and 'h' is thickness of the plate. The gradation of material properties is in the transverse direction with metal and ceramic being the typical constituents. Aluminum/Alumina (Al/Al_2O_3) and Aluminum/Zirconia (Al/ZrO_2) are the examples of the functionally graded plate.

2.1. Material variation laws

The constituent elements of FGP are varying in transverse direction from bottom, where it is metal rich to the top, where the surface is ceramic rich. Macroscopically the plate is assumed homogenous and isotropic. This variation is achieved by changing the volume fraction of the constituent elements. The volume fraction and hence material properties may vary according to exponential law (Zenkour [2]) or power law (Reddy [16]). For the exponential law the material properties vary exponentially from bottom to top and properties at any section z are represented in terms of properties at the bottom surface as given by Eq. (1).

$$P_e = P_0 e^{N(z+1/2)} \tag{1}$$

where, P_e represents effective material properties at any section across thickness, P_0 represents the property at bottom surface; for our study, $P_0 = P_m$ where P_m represents property of metal. N is the exponent on which the total variation of properties depends known as power index. As N increases the properties change drastically from bottom to top and this variation across the thickness for

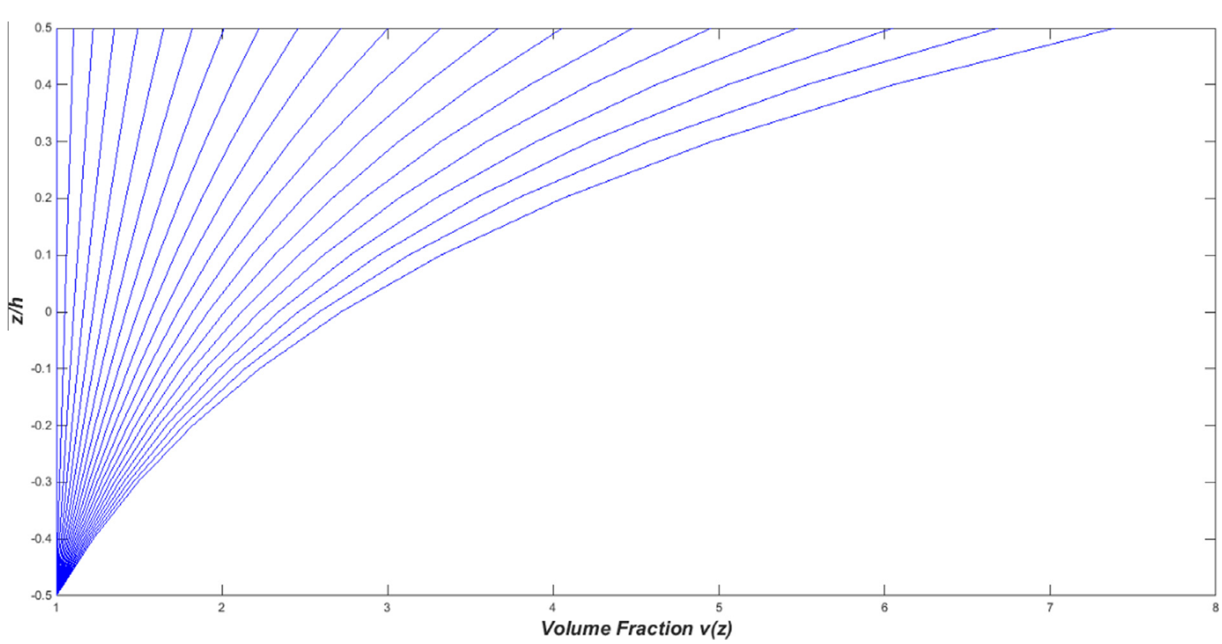

Fig. 2. Volume fraction variation by exponential law.