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Bending deflection of sandwich beams considering local effect of concentrated force

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ABSTRACT

The sandwich constructions are widely used in various industrial applications due to their ability to provide high bending stiffness coupled with light weight. In general, the sandwich structures are designed as panels that must carry bending loads. The three-point bending test is one of the most common material testing procedures for investigating the properties and behavior of materials with structural applications. In this sense, the bending deflection is one of the properties investigated by bending tests and is associated with the stiffness of the analyzed material.

Considering this, within this paper is proposed a correction to first and Reddy's third order shear deformation theory to determine the bending deflection of sandwich beams. The method is based on the Timoshenko's theory for highlighting the local effect given by the applied load, adapted for sandwich structures. Also, this method approach the bending deflection maintaining compact the sandwich beam, is easy to apply and accurately estimate the vertical displacements along the cross-section. Two sandwich configurations were analyzed in the paper. The results showed on the one hand, the capability of the proposed relations to accurately estimate the vertical displacements of different points on the cross section and on the other hand the core flexibility effect on the bending behavior of the analyzed sandwich beams.

1. Introduction

The sandwich structures have been shown to be the most effective elements for strength and stiffening, and this is shown by the numerous industrial applications and respective designing configurations. Besides the strength, stiffness and strain energy absorption capacity are other basic characteristics of sandwich structures.

In general the sandwich structures are designed as panels that carry bending loads. The bending stiffness is characterized by the slope of the variation curve between the applied force and the vertical displacement of the point of application of force. This vertical displacement is often associated with the vertical deflection of the sandwich structure. In this regard, based on the Timoshenko beam theory, Allen [1] proposed the following relation for the total deflection, δ, at the mid-section of a sandwich beam loaded in 3-point bending:

$$\delta = \frac{Fl^3}{48 \cdot EI_{eq}} + \frac{Fl}{4 \cdot GA_{eq}} \tag{1}$$

where EI_{eq} is the equivalent bending stiffness of the sandwich beam and GA_{eq} is the equivalent shear stiffness of the sandwich beam and l is the span length.

Eq. (1) was generally accepted and assumes that in the loading plane the vertical deflection of the neutral axis is the same as the vertical displacement of the point of application of force. In reality there are very few cases in which this condition is valid. In the case of bending of sandwich beams with flexible cores (e.g. foam, cork) and thin faces, the variation curve of the applied force and vertical deflection can be easily affected by the occurrence of some local effects such as indentation or core damage.

Rubino et al. [2] have conducted an experimental study on three-point bending of *Y*-frame and corrugated core sandwich beams. They showed that the indentation of the core is the main contributor to the displacement, δ, of the loading point and proposed an analytical upper bound model for the indentation response of the sandwich beams. The energy absorption capacity of empty and aluminum foam-filled beams with corrugated cores have been experimentally and analytically studied by Yan et al. [3]. Failure mechanism maps for three-point bending were constructed for both empty and filled sandwich beams and the bending stiffness, initial failure load and peak load were predicted by theoretical analysis. The results have shown that filling of

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aluminum foam into the interstices of corrugated core changed dramatically the force versus displacement curve of the sandwich beam, enhancing significantly its load-bearing capacity. During the 3-point bending tests the force and displacement of the specimens were measured by the loading cell of the test machine. Also the support and the head of the loading device were steel cylinders with a flat pedestal attached to avoid force concentration. Conducting an experimental study on three-point bending of sandwich beams with aluminum foam and thin faces, Kabir et al. [4] have shown a significant indentation effect in the sandwich beam beneath the center loading roller characterized by local damage of the foam core.

Different approaches and analytical models have been developed in order to accurately describe the deformation fields in sandwich structures and to predict the deflection under different loading conditions.

Frostig et al. [5] and Frostig [6] developed a theory based on variational principle for sandwich beams with flexible core, named High Order Sandwich Panel Theory (HSAPT). In this theory the faces are modeled as two beams interconnecting with a core considered as a two-dimensional elastic medium. The nonlinear displacement fields through the depth of the core were determined. The implementation of this theory was demonstrated in several numerical examples that dealt with concentrated loads. The effect of concentrated load is associated with the vertical flexibility of the core and also the theory is sensitive to the point of load application; a different behavior is detected for a load applied at the upper face than at the lower face. Based on the HSAPT, Dariushi and Sadighi [7] have developed a geometrically nonlinear high order sandwich beam theory that considers large strains of face sheets and core. The mathematical formulations are derived using Green strain and second Piola–Kirchhoff stress tensor and were applied for a simply supported sandwich beam under local transverse load. The results of this method were compared with linear HSAPT and have showed that the difference between predicted deflections by nonlinear model and that by linear model increases by increasing beam length and decreases with increasing face sheet and core thickness.

Considering the couple stress effects, Romanoff and Reddy [8] have developed an analytical model for prediction of the deflection of a web core sandwich beam loaded in 3 and 4-point bending. In this model a length scale parameter is considered which is equal to the web core cell size and the predictions of deflection have good accuracy compared to that obtained by Eq. (1).

In this study was analyzed the local effect given by the applied force on the bending deflection of sandwich beams. The local effect of the applied concentrated force is considered as the solution given by Timoshenko for these problems [9], and to determine the bending deflection considering local effects have been used the first and Reddy's third-order shear deformation theory, [10] adapted for beams in bending.

2. Material and methods

2.1. Sandwich beam loaded by a concentrated force

According to elasticity theory of Timoshenko, near the point of application of concentrated force, a serious local perturbation in stress distribution should be expected and the problem must be considered for analysis. To find the solution of this problem, it is considered a concentrated vertical force P applied on a horizontal straight boundary AB of an infinitely large plate, Fig. 1(a). The distribution of the load along the thickness of the plate is uniform as indicated in Fig. 1(b). The thickness of the plate is taken as unity. Around the point of application of force P, the stress has a simple radial distribution. Any point A at a distance r from the point of application of the load is subjected to a simple compression in the radial direction and the radial stress is given by:

$$\sigma_r = -\frac{2P}{\pi}\cdot\frac{\cos\theta}{r} \tag{2}$$

$$r = d\cdot\cos\theta \tag{3}$$

The tangential stress σ_θ and the shearing stress $\tau_{r\theta}$ are zero. Also, the boundary conditions are satisfied because σ_θ and $\tau_{r\theta}$ are zero along the straight edge of the plate, which is free from external forces except at the point of application of the load. Here σ_r becomes infinite. The resultant of the forces acting on a cylindrical surface of radius r (Fig. 1(b)) must balance P. This is obtained by summing the vertical components $\sigma_r\cdot r\cdot d\theta$ acting on each element $r\cdot d\theta$ of the surface:

$$2\int_0^{\frac{\pi}{2}}\sigma_r\cos\theta\cdot rd\theta = -\frac{4P}{\pi}\int_0^{\frac{\pi}{2}}\cos^2\theta d\theta = -P \tag{4}$$

In the case of a simply supported sandwich beam loaded in three-point bending by a concentrated force P applied at the mid-span (Fig. 2(a)), the total stress along the section AB, can be found by the superposition of the bending stress distribution and the stress created by the line load, given by Eq. (2) for a semi-infinite plate.

The radial pressure distribution created by the applied load over quadrant ab of the cylindrical surface abc at point A (Fig. 2(b)) produce a horizontal and vertical force given by the relations (5) and (6), applied to A (Fig. 2(c)).

$$\int_0^{\frac{\pi}{2}}(\sigma_r\sin\theta)rd\theta = \int_0^{\frac{\pi}{2}}\frac{2P}{\pi}\sin\theta\cdot\cos\theta d\theta = \frac{P}{\pi} \tag{5}$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(\sigma_r\sin\theta)rd\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{2P}{\pi}\cos^2\theta d\theta = P \tag{6}$$

Thus, the bending moment about the point O is:

$$M_o = \frac{PL}{4} - \frac{P}{\pi}c \tag{7}$$

In these circumstances, the law of the bending moment variation for the entire beam is written in the following form:

$$M_{xx} = \left(\frac{P}{2} - \frac{2Pc}{\pi L}\right)x \tag{8}$$

For $x = 0$ in Fig. 3, it results the bending moment $M_1 = 0$ and for $x = L/2$ results the bending moment M_o. The relation (8) is valid for the entire beam if the external applied force to the beam is equivalent to $P - \frac{4Pc}{\pi L}$. This means that when a concentrated force P is applied on a beam in three-point bending, its effect on the beam is equivalent to one given by a load equal to $P - \frac{4Pc}{\pi L}$.

2.2. Bending deflection of sandwich beam by first-order shear deformation theory (FSDT)

The displacement field from a beam in bending, according to first-order shear deformation theory, is of the form, [11]:

$$u(x,z) = u_0(x) + z\phi_x(x) \tag{9}$$

$$w(x,z) = w_0(x) \tag{10}$$

where ϕ_x denotes the rotation of the cross section about y-axis (normal on the x and z axes), u_0 and w_0 are the displacements of a point on the plane $z = 0$.

The linear strains associated with the displacement field are as follows: