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Vibration of thin-walled laminated composite beams having open and closed sections

Abdul Hamid Sheikh [a,*], Arash Asadi [a], Ole Thybo Thomsen [b,c]

[a] School of Civil, Environmental and Mining Engineering, The University of Adelaide, Adelaide, Australia
[b] Engineering and the Environment, University of Southampton, Highfield, Southampton, United Kingdom
[c] Department of Mechanical and Manufacturing, Aalborg University, Aalborg, Denmark

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ABSTRACT

An efficient technique based on one dimensional beam finite element analysis for vibration of thin-walled laminated composite beams having open and closed sections is proposed in this paper. The developed technique is quite generic which can accommodate any stacking sequence of individual walls and considers all possible couplings between different modes of deformation. The formulation has accommodated the effect of transverse shear deformation of walls as well as out of plane warping of the beam section where the warping can be restrained or released. The inclusion of shear deformation has imposed a problem in the finite element formulation of the beam which is solved successfully utilising a concept developed by one of the authors. A number of numerical examples of open section (I and C sections) beams and closed section box beams are solved by the proposed technique and the results predicted by the proposed model are compared with those obtained from literature as well as detailed finite element analysis using a commercial code. The results show a very good performance of the proposed modelling technique.

1. Introduction

The use of thin-walled beam like slender structures made of laminated composite materials is found in many engineering applications such as helicopter rotor blades, construction industry, long wind turbine blades and few other situations. The behavior of these structures can be accurately predicted by a detailed finite element model using three-dimensional (3D) or shell elements but the computational demand of such model is extremely high. In order to avoid this problem, a group of researchers tried to model these structures as a condensed one-dimensional (1D) beam elements which will drastically improve the computational efficiency but the major challenge in that approach is the formulation of a suitable beam element that will be able to capture all effects and their couplings found in these complex thin-walled composite structural system. This has drawn attention of a number of researchers which made this topic an active area of research in recent years. Some representative samples of these models are provided in references [1–8]. The studies carried out so far can be divided into two broad groups based on the technique used to determine the cross-sectional matrixes which can be used to formulate the one dimensional beam element. The first approach is based on 'analytical techniques' while the second approach uses a two-dimensional (2D) cross-sectional analysis based on a 2D finite element model for the determination of the cross-sectional matrixes which can be utilised to carry out the 1D beam analysis.

Hodges and his co-workers [3] have significantly contributed toward the development of the second approach where the three dimensional (3D) elasticity problem defining the deformation of these beam like structures is systematically divided into a one-dimensional (1D) beam problem and a 2D cross-sectional problem. This method is generally referred to as variational asymptotic beam section analysis (VABS) which is based on variational asymptotic method (VAM) [9]. This approach is also suitable for modelling solid and the thick walled cross-sections. The same group of researchers ([10–14]) has also attempted to solve the 2D cross-sectional problem defined within the framework of VAM analytically but this approach involves rigorous mathematical treatments to evaluate the cross-sectional stiffness coefficients.

Since the present paper is primarily focusing on the analysis of thin-walled composite beams, the analytical approach (first approach) is used for determination of cross-sectional matrixes. Moreover, the 2D finite element analysis or a complex mathematical treatment involved with the other approach is avoided in the present investigation. Specifically, the current study has adopted

* Corresponding author.
E-mail address: abdul.sheikh@adelaide.edu.au (A.H. Sheikh).

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the analytical approach proposed by Sheikh and Thomsen [8] and extended to vibration. The different modes of deformation considered in the development of present closed-form analytical solution are axial, torsion, bi-axial bending, bi-axial shear as well as out of plane warping for the torsional deformation. The cross-sectional matrices are explicitly derived for the open I section, C section and closed box section. The present formulation considered both plane stress and plain strain conditions of a lamina. The 1D beam problem is solved by the finite element approximations. The incorporation of the out of plane warping displacement demands a C^1 continuous finite element formulation for the twisting rotation, which is accommodated using Hermetian interpolation functions. On the other hand, the usual treatment of the transverse shear deformation requires a C^0 formulation. This needs the use of reduced integration technique for avoiding any shear locking problem. The different degrees of continuity for the different modes of deformation and their coupling impose a problem for their numerical implementation. This problem is addressed satisfactorily utilising the concept proposed by Sheikh [15] which permits a full integration and avoid the problem of using any reduced integration. For the 1D beam finite element analysis, a three node beam element as shown in Fig. 1 has been developed.

A computer code is written in FORTRAN to implement the present formulation. Numerical examples of thin-walled composite beams having different cross sections and other conditions are analysed by the proposed model and the results obtained in the form of vibration frequencies are validated with the available results in literature. These results demonstrate a very good performance of proposed model.

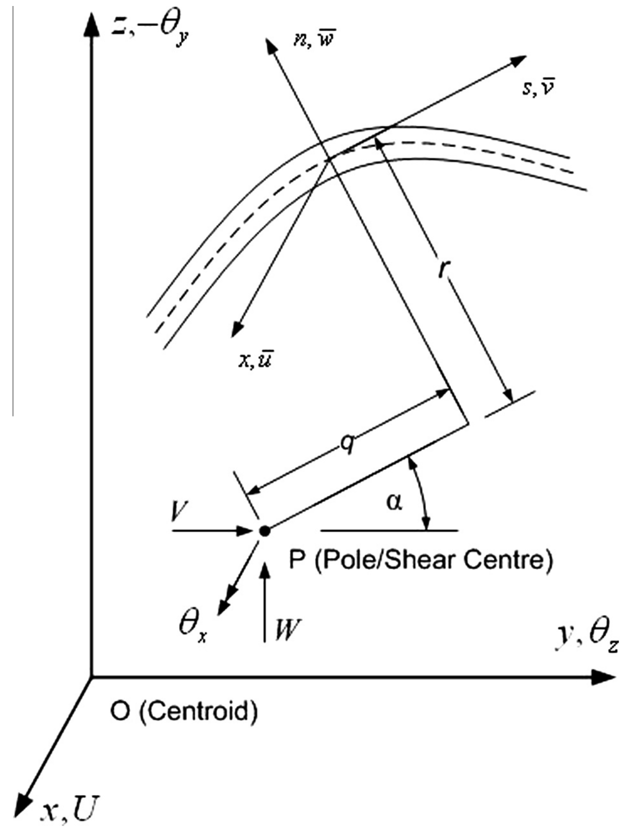

Fig. 2. Cross-section of a portion of beam shell wall with local and global coordinate system and displacement components.

2. Formulation

Fig. 2 shows a segment of the composite beam shell wall where x–y–z is taken as the global Cartesian coordinate system with x being directed along the beam axis which is passing through the centroid of the beam section. A local orthogonal coordinate system x–s–n is also defined where x–s plane passes through the tangential plane of beam wall mid-plane (local x-axis is parallel to the global x-axis) and n is directed along the wall thickness. The displacement components at the mid-plane of the shell wall in the local coordinate system (x–s–n) can be expressed in term of the global displacement components of the beam [1] as

$$\begin{aligned}\bar{u} &= U + y\theta_y + z\theta_z + \varphi\theta'_x,\\ \bar{v} &= V\cos\alpha + W\sin\alpha - r\theta_x,\\ \bar{w} &= -V\sin\alpha + W\cos\alpha + q\theta_x,\end{aligned} \tag{1}$$

where φ is the warping function, θ_x is the torsional rotation and θ_y, θ_z are bending rotations of the cross-section of the beam along (not about) y and z, respectively. These bending rotations can be expressed as $\theta_y = -V' + \Psi_y$ and $\theta_z = -W' + \Psi_z$, where Ψ_y, Ψ_z are shear rotations of the beam section about z and y, respectively, and V', W' and θ'_x are respectively the derivatives of V, W and θ_x with respect to x.

It has been observed that the warping displacement of a closed section beam is relatively less than that of an open section beam [14] but the present formulation has considered the effect of warping in all cases. Considering the effects of bending and transverse shear deformation of the beam shell wall, the displacements at any point of the shell wall away from its mid-plane may be expressed as

$$\begin{aligned}u &= \bar{u} + n\left(-\tfrac{\partial\bar{w}}{\partial x} + \psi_{xn}\right),\\ v &= \bar{v} + n\left(-\tfrac{\partial\bar{w}}{\partial s} + \psi_{sn}\right),\\ w &= \bar{w},\end{aligned} \tag{2}$$

where ψ_{xn} and ψ_{sn} are shear rotations of the shell wall section about s and x, respectively. It is assumed that $\psi_{sn} = 0$ whereas ψ_{xn} can be expresses in terms of the corresponding global components (Ψ_y and Ψ_z) as $\psi_{xn} = -\Psi_y\sin\alpha + \Psi_z\cos\alpha$. Substituting this as well as Eq. (1) in the above Eq. (2), the displacements at any point within the shell wall along its local coordinate system (x–s–n) can be expressed in terms of the global displacement components of the 1D beam as

$$\begin{aligned}u &= U + (y - n\sin\alpha)\theta_y + (z + n\cos\alpha)\theta_z + (\varphi - nq)\theta'_x,\\ v &= V\cos\alpha + W\sin\alpha - (r+n)\theta_x,\\ w &= -V\sin\alpha + W\cos\alpha + q\theta_x.\end{aligned} \tag{3}$$

With the above beam kinematics (3), the free vibration governing equation for the beam can be derived from its total energy. The total energy of a structure consists of strain energy (U) and kinetic energy (T) which can be used to derive the stiffness matrix $[K]$ and mass matrix[M], respectively used in finite element analysis of the structure. As the derivation of the stiffness matrix $[K]$ has already been shown elsewhere [8], it is not repeated here. Taking ρ as density of the material, the kinetic energy for free vibration of a beam may be written as

$$T = \frac{1}{2}\int_V \{\dot{U}\}^T \rho \{\dot{U}\} dv = -\frac{\omega^2}{2}\int_V \{U\}^T \rho \{U\} dv. \tag{4}$$

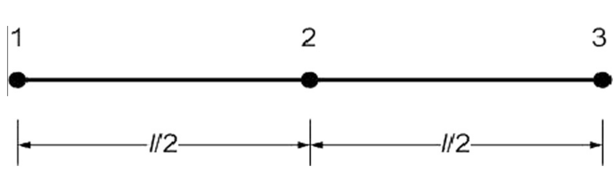

Fig. 1. A typical beam element.