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Interfacial stresses in sandwich structures subjected to temperature and mechanical loads

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ABSTRACT

Through sequential coupling of the differential equations for interfacial shearing and peeling stresses, concise closed-form strength-of-material solutions for these stresses in symmetric and non-symmetric sandwich structures subjected to the combined loadings of temperature, stretching, and bending have been developed. The free-edge effects of the interfacial shearing stress, the maximum magnitude of the interfacial shear stress, and the location of its occurrence are accurately modelled using a single high-frequency hyperbolic function. The analytical solutions have been validated against finite element analysis (FEA) solutions using four sandwich structures: one symmetric, two mildly asymmetric, and another severely asymmetric; and for three load cases: differential thermal expansion, differential mechanical stretching, and mechanical bending. The maximum deviation between the analytical and the FEA solutions for the interfacial shearing and peeling stresses for all the structures and load cases, including that for the severely asymmetric sandwich structure, was less than 15%.

Mathematical symbols

$\alpha_z^4 = \frac{\bar{D}_e}{4\lambda_z}$
$\beta^2 = \frac{\lambda_x}{\kappa_s}$
$C_b = \frac{M}{\lambda_z \alpha_z^2 D_2}$
$C_s = \frac{\mu^* \alpha_{21} \Delta T}{\lambda_z \kappa_s \left(4\alpha_z^4 + \beta^4\right)}$
$D_i = \frac{E_i h_i^3}{12}$, $\bar{D}_i = \frac{1}{D_i}$
$\bar{D}_e = \bar{D}_1 + \bar{D}_2$
$\mu = \frac{1}{2}\left(\frac{h_2}{D_2} - \frac{h_1}{D_1}\right)$, $\mu^* = \frac{1}{2}\left(\frac{h_2+h_3}{D_2} - \frac{h_1+h_3}{D_1}\right)$
$\kappa_s = \kappa_{s1} + \kappa_{s2} + \kappa_{s3}$
$\kappa_{si} \approx \frac{h_i}{8G_i}$, $\kappa_{s3} = \frac{h_3}{G_3}$
$\lambda_x = \lambda_{x1} + \lambda_{x2} + \lambda_{x\theta}$
$\lambda_{xi} \approx \frac{1}{E_i h_i}$
$\lambda_{x\theta} = \frac{1}{4}\left(\frac{h_1^2 + h_1 h_3}{D_1} + \frac{h_2^2 + h_2 h_3}{D_2}\right)$
$\lambda_z = \lambda_{z1} + \lambda_{z2} + \lambda_3$
$\lambda_{zi} \approx \frac{3h_i}{8E_i}$, $\lambda_{z3} = \frac{h_3}{E_3}$
Note: The above formulae involving E_i are for plane stress; substitutes E_i with $E_i' = E_i/(1-\nu_i^2)$ for plane strain

1. Introduction

Many engineering structures are made of sandwich construction. A sandwich structure in the aircraft, the ship-building, the wind-energy, and the civil construction industries is composed of two thin stiff facesheets bonded by a thick lightweight core. On the other hand, a sandwich structure in the microelectronic and optoelectronic industries is made of an integrated circuit (IC) component that is electrically interconnected and mechanically bonded to a printed circuit board (PCB) which is made of weaved glass-fibre reinforce (FRP) polymers interspaced with copper foils that are inscribed with networks of electrical circuitries. The bonding layer is typically made of tiny discrete solder joints. But for portable electronics, which is susceptible to drop-impact, the solder joints are typically reinforced with polymeric adhesive that fills the space between the discrete solder joints.

When in operation, the IC component is heated up by the millions of operating transistors and the differential coefficient of thermal expansion between the IC component and the PCB gives rise to mismatched thermal expansion. Portable electronics are susceptible to drop-impact. During drop-impact, the PCB undergoes flexural deformation as well as membrane stretching while the IC component does not [1]. The mismatched thermal expansion and the differential bending and stretching deformations between the IC component and the PCB must be accommodated by the

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Nomenclature

Subscript $\#_i$	subscript #1 and #2 are outer members and #3 is a bonding layer
D_i, E_i, G_i, h_i	flexural rigidity, elastic modulus, shear modulus, thickness of member #i
$\bar{D}_i$, $\bar{D}_e$	flexural compliance of member #i, effective flexural compliance of the sandwich structure
f_a, f_b	mean, amplitude of the interfacial peeling stresses
f_s	interfacial shear stress
F_i	x-directional sectional traction acting along the centroid axis of member #i
L	half length of the sandwich structure
M_i	sectional moment on member #i
Q_a, Q_b	sectional shear forces corresponding to f_a and f_b, respectively
u_i, w_i	x-directional, z-directional displacement of the centroid axis of member #i
α_z, β	characteristic constants for through-thickness displacement, shear deformation of the sandwich structure
α_i; α_{ij}	coefficient of thermal expansion of member #i; differential coefficient of thermal expansion between member #i and member #j
κ_{si}, κ_s	shear compliances of member #i, of the sandwich structure between the centroid axes of members #1 and #2
λ_{xi}, $\lambda_{x\theta}$, λ_x	x-direction stretch compliances of member #i, of contribution due to the bending rotation of members, of the sandwich structure
λ_{zi}, λ_z	through-thickness compliances of member #i, of the sandwich structure
μ, μ^*	parameters that describe the differential bending compliances of members #2 and #1
θ_i	rotation of the centroid axis of member #i due to bending
ϕ_3, $\bar{\phi}_i$	rotation of member #3, average rotation of the cross-section of member #i w.r.t. their respective centroid axis due to shearing
ΔT	temperature change

bonding layer subjecting it to extreme stresses. The main concern has always been the delamination of the bonding interfaces, which is typically initiated from the free edges.

Historically, the solutions for the thermal and the mechanical loadings have developed separately. The pioneering work in analysing interfacial stresses due to mismatched thermal expansion of layered structures made of dissimilar materials could be traced to Aleck [2] and this is followed by rather rich publications driven principally by the microelectronic community. The analysis have taken to two separate approaches: the theory-of-elasticity approach and the strength-of-material approach. In the theory-of-elasticity approach, the assembly is modelled as an elastic continuum domain and stress functions of varying complexity are assumed within the domain. Examples of this approach include Aleck [2], Boley and Testa [3], Hess [4], Chen et al. [5], Kuo [6], Yin [7], Lee and Jasiuk [8], and Feng and Wu [9]. In the strength-of-material approach, the outer layers are modelled as beams while the bonding layer is modelled as distributed infinitesimal springs. Examples of this approach include Taylor and Yuan [10], Grimado [11], Chen and Nelson [12], Willams [13], Suhir [14,15], Pao and Eisele [16], Jiang et al. [17], Ru [18], Wen and Basaran [19], and Wong et al. [20,21]. Further references may be found in the review article of Suhir [22].

The first reported analysis of layered structures subjected to mechanical loading could be traced to Volkersen [23]. This is followed by the classical work of Goland and Reissner [24]. Since then, the analyses have also taken to the two separate approaches of the theory-of-elasticity and the strength-of-material. Examples of the theory-of-elasticity approach are Goland and Reissner in modelling rigid bond [24], Pirvics [25], Chen and Cheng [26], Cheng et al. [27], Adams and Mallick [28], Sawa et al. [29], Lovinger and Frostig [30] and Wu and Zhao [31]. Examples of the strength-of-material approach are Goland and Reissner in modelling flexible bond [24], Allman [32], Renton and Vinson [33], Ojalvo and Eidinoff [34], Delale et al. [35], Bigwood and Crocombe [36], Frostig [37], Tsai et al. [38] and Wang and Zheng [39]. Further references may be found in the review article of Silva [40].

While attempts have been made to integrate the strength-of-material solutions for the thermal and the mechanical loadings [41], the results have not been satisfactory. Moreover, except for the solutions of Renton and Vinson [33], Suhir [15], Ru [18], and Wang and Zheng [39], who uses differential equations of between six and eight orders, the solutions are unable to model the free-edge effects for the interfacial shearing and peeling stresses, missing at where it is most critical. On the other hand, the solutions derived from the high-order differential equations are far too obscured for insights and too complex for adoption by practising engineers.

This article presents a strength-of-material solution for the interfacial stress in a sandwich structure subjected to a combination of mismatched thermal expansion, differential free-edge stretching, and free-edge bending that resulted in the symmetric deformation of the sandwich structure about its mid-length. The free-edge condition for the interfacial shearing stress is modelled using a variable-frequency hyperbolic function leading to a concise closed-form solution. The solutions are validated with finite element analysis (FEA) using symmetric and severely non-symmetric sandwich structures.

2. The Fundamental Equations

Consider a sandwich structure experiencing a uniform temperature rise of ΔT and subjected to sectional traction F and sectional moment M applied to the free edges of its outer members such that the sandwich structure deforms symmetrically about its mid-length. Fig. 1 shows the elemental representations of the sandwich structure, wherein the structural members #1 and #2 are modelled as beam element while the bonding layer, member #3, is modelled as a two-dimensional elastic body that has negligible stiffness in the x-direction. Designating the bonding interface between members #2 and #3 as the top interface and that between members #1 and #3 as the bottom interface, the interfaces experience an equal magnitude of shear stress, f_s, but different magnitudes of peeling stress, f_p. The assumption of negligible x-directional stiffness of the bonding layer leads to a linearly varying peeling stress over its thickness with a mean magnitude, f_a, and an amplitude of variation, f_b. The peeling stress at the two interfaces are, respectively,

$$\begin{aligned} f_{p,23} &= f_a + f_b, \\ f_{p,13} &= f_a - f_b. \end{aligned} \tag{1}$$