Composite Structures 134 (2015) 237–246

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

# A beam formulation for large displacement analysis of composite frames with semi-rigid connections

Goran Turkalj *, Domagoj Lanc, Josip Brnic, Igor Pesic

*Department of Engineering Mechanics, Faculty of Engineering, University of Rijeka, Vukovarska 58, HR-51000 Rijeka, Croatia*

ARTICLE INFO

*Article history:*
Available online 24 August 2015

*Keywords:*
Composite frames
Large displacement
Semi-rigid connections
Beam element

ABSTRACT

This paper presents a geometrically non-linear beam formulation for large displacement analysis of composite beam type structures with semi-rigid connections. Within the framework of updated Lagrangian incremental formulation and the nonlinear displacement field of thin-walled cross-sections, which accounts for restrained warping and the second-order displacement terms due to large rotations, the equilibrium equations of a straight beam element are firstly developed. Due to the nonlinear displacement field, the geometric potential of semitangential moment is obtained for both the internal torsion and bending moments, respectively. To account for the semi-rigid connection behaviour, a special transformation procedure is developed. The laminates are modelled on the basis of classical lamination theory. In order to illustrate the application of the proposed formulation, several numerical examples are presented.



## 1. Introduction

The interest for using fibre reinforced composites in construction applications in recent years increased rapidly. Composite structural beam members have been introduced to replace some of the conventional materials, primary in aerospace industry and in past few decades also in civil engineering applications [1,2]. Commercially, the beam members are produced in a number of different "steel-like" profiles including H-, I-, L- and tubular profiles, imitating traditional structural members. Such structures generally could display very complex structural behaviour. Due to their slenderness and specific mechanical properties, they are commonly very susceptive to instability or buckling failure.

Many papers have been devoted to finite element buckling analysis of different types of composite beams and only some of them are cited here [3–10]. In these papers, different theories (classical beam theory and higher-order beam theory) have been introduced. However, there are quite a few papers dedicated to problems of flexibility of the framing connections such as beam-column, beam-girder and column-base connections. As is the case of steel frames, joint flexibility may have a very significant influence on the global behaviour of beam structure, and therefore, the joint flexibility of composite frames remains to be devoted to lot of attention [11].

The author's previous papers [12,13] are devoted to flexibly connected isotropic beam-type structures as well as to simulation of laminated composite frames with no attention to connection flexibilities. In this paper, which is a synthesis of the previous two, the large displacement analysis of composite frames with semi-rigid connections is presented.

The model is based on assumptions of large displacements but small strains, the Euler–Bernoully beam theory for bending and the Vlasov theory for torsion. The thin-walled beam members are supposed to be straight and prismatic and the classical lamination theory is adopted. External loads are assumed to be static and conservative. In order to perform non-linear stability analysis in load deflection manner, the updated Lagrangian (UL) incremental descriptions is applied. The non-linear cross section displacement field which accounts for the second order displacement terms due to large rotations is implemented. The generalised displacement control method [14] is employed in terms of the incremental–iterative solution scheme, and updating of nodal orientations at the end of the each iteration is performed using the transformation rule which applies for semitangental incremental rotations [15]. The force recovering is performed according to the conventional approach (CA) [12,16].

A hybrid element, hereafter called the SR element, composed of the aforementioned nonlinear beam element and dimensionless linear/nonlinear springs added at element nodes is introduced for modelling the structures at which flexible connections may occur. One side of each spring is connected to a node of the beam

* Corresponding author. Tel.: +385 51 651 499; fax: +385 51 651 490.
*E-mail address:* goran.turkalj@riteh.hr (G. Turkalj).

http://dx.doi.org/10.1016/j.compstruct.2015.08.068
0263-8223/

element, while the other side is connected to a global node. Using the SR beam element, connections are no longer assumed to be fully rigid [17–19].

## 2. Basic consideration

### 2.1. Beam kinematics

In this paper, two sets of coordinate systems, which are mutually interrelated, are used. The first coordinate system is Cartesian coordinate system $(z,x,y)$, for which $z$-axis coincides with the beam axis passing through the centroid O of each cross-section, while the $x$- and $y$-axes are the principal inertial axes of the beam. The second coordinate system is contour coordinate $(z,n,s)$ as shown in Fig. 1, wherein coordinate $z$ coincident with beam $z$-axis, the coordinate $s$ is measured along the tangent of the middle surface in a counter-clockwise direction, while $n$ is the coordinate perpendicular to $s$.

Incremental displacement measures of a cross-section are defined as

$$\begin{aligned} & w_{\mathrm{O}} = w_{\mathrm{O}}(z); \quad u_{\mathrm{S}} = u_{\mathrm{S}}(z); \quad v_{\mathrm{S}} = v_{\mathrm{S}}(z); \quad \varphi_z = \varphi_z(z) \\ & \varphi_x = -\frac{\mathrm{d}v_{\mathrm{S}}}{\mathrm{d}z} = \varphi_x(z); \quad \varphi_y = \frac{\mathrm{d}u_{\mathrm{S}}}{\mathrm{d}z} = \varphi_z(z); \quad \theta = -\frac{\mathrm{d}\varphi_z}{\mathrm{d}z} = \theta(z) \end{aligned} \tag{1}$$

where $w_{\mathrm{O}}$, $u_{\mathrm{S}}$ and $v_{\mathrm{S}}$ are the rigid-body translations of the cross-section associated with the centroid in the $z$-direction and the shear centre in the $x$- and $y$-directions; $\varphi_z$, $\varphi_x$ and $\varphi_y$ are the rigid-body rotations about the shear centre $z$-, $x$- and $y$-axis, respectively; $\theta$ is a parameter defining the warping of the cross-section. The superscript 'prime' indicates the derivative with respect to $z$.

Let $\mathbf{r}_{\mathrm{O}}$ denotes the position vector of a material point in the reference configuration and $\mathbf{U}_{\mathrm{O}}$ the translation displacement vector of the centroid. If the assumption of small rotations is valid, then the incremental displacement field $\mathbf{U}_{\mathrm{LDF}}$, containing the first-order displacement increments of an arbitrary point on the cross-section defined by the position coordinates $x$ and $y$ and the warping function $\omega(x,y)$, can be written in the following form [20]:

$$\mathbf{U}_{\mathrm{LDF}} = \mathbf{U}_{\mathrm{O}} + \tilde{\boldsymbol{\varphi}}\mathbf{r}_{\mathrm{O}} - \tilde{\boldsymbol{\omega}}\mathbf{s} \tag{2}$$

where

$$\mathbf{U}_{\mathrm{LDF}} = \begin{Bmatrix} u_z \\ u_x \\ u_y \end{Bmatrix}; \quad \mathbf{U}_{\mathrm{O}} = \begin{Bmatrix} w_{\mathrm{O}} + \omega\theta \\ u_{\mathrm{S}} \\ v_{\mathrm{S}} \end{Bmatrix}; \quad \mathbf{r}_{\mathrm{O}} = \begin{Bmatrix} 0 \\ x \\ y \end{Bmatrix}; \quad \mathbf{s} = \begin{Bmatrix} 0 \\ x_{\mathrm{S}} \\ y_{\mathrm{S}} \end{Bmatrix}$$
$$\tilde{\boldsymbol{\varphi}} = \begin{bmatrix} 0 & -\varphi_y & \varphi_x \\ \varphi_y & 0 & -\varphi_z \\ -\varphi_x & \varphi_z & 0 \end{bmatrix}; \quad \tilde{\boldsymbol{\omega}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\varphi_z \\ 0 & \varphi_z & 0 \end{bmatrix} \tag{3}$$

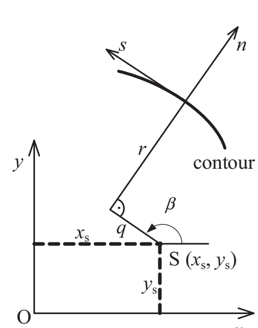


**Fig. 1.** Contour coordinate system.

If the assumption of small rotations is invalid, i.e. if the large rotation effects are taken into account, then the non-linear incremental displacement field $\mathbf{U}_{\mathrm{NDF}}$ should be introduced in the analysis, i.e. [21]

$$\mathbf{U}_{\mathrm{NDF}} = \mathbf{U}_{\mathrm{LDF}} + \tilde{\mathbf{U}}; \quad \tilde{\mathbf{U}} = \{\, \tilde{u}_z \quad \tilde{u}_x \quad \tilde{u}_y \,\}^{\mathrm{T}} = 0.5(\tilde{\boldsymbol{\varphi}}^2\mathbf{r}_0 + \tilde{\boldsymbol{\varphi}}\tilde{\boldsymbol{\omega}}\mathbf{s}) \tag{4}$$

in which $\tilde{\mathbf{U}}$ contains the additional or second-order displacement increments due to large rotations.

According to the non-linear displacement field given by Eq. (3), the Green–Lagrange incremental strain tensor can be written as:

$$\varepsilon_{ij} = 0.5\left[(u_i+\tilde{u}_i)_{,j} + (u_j+\tilde{u}_j)_{,i} + (u_k+\tilde{u}_k)_{,i}(u_k+\tilde{u}_k)_{,j}\right] \cong e_{ij} + \eta_{ij} + \tilde{e}_{ij} \tag{5}$$

where:

$$e_{ij} = 0.5(u_{i,j} + u_{j,i}); \quad \eta_{ij} = 0.5u_{k,i}u_{k,j}; \quad \tilde{e}_{ij} = 0.5(\tilde{u}_{i,j} + \tilde{u}_{j,i}) \tag{6}$$

It should be noted here that according to the geometrical hypothesis of the in-plane rigidity for the cross-section, the strain components $\varepsilon_{xx}$, $\varepsilon_{yy}$ and $\gamma_{xy} = 2\varepsilon_{xy}$ in Eq. (6) should be equal to zero.

### 2.2. Contour displacements

The contour mid-line displacements are: $\bar{w}$, $\bar{u}$ and $\bar{v}$, while the out of mid-line displacement components are defined as:

$$w(z,s,n) = \bar{w} - n\frac{\partial\bar{u}}{\partial z}; \quad v(z,s,n) = \bar{v} - n\frac{\partial\bar{u}}{\partial s}; \quad u(z,s,n) = \bar{u} \tag{7}$$

The beam-to-contour displacement relation can be written as:

$$\begin{aligned} & \bar{w}^{\mathrm{L}} = u_z(z,s,n); \quad \bar{w}^{\mathrm{NL}} = \tilde{u}_z(z,s,n); \\ & \bar{v}^{\mathrm{L}} = u_x(z,s,n)\cos\beta + u_y(z,s,n)\sin\beta; \\ & \bar{v}^{\mathrm{NL}} = \tilde{u}_x(z,s,n)\cos\beta + \tilde{u}_y(z,s,n)\sin\beta; \\ & \bar{u}^{\mathrm{L}} = u_x(z,s,n)\sin\beta - u_y(z,s,n)\cos\beta; \\ & \bar{u}^{\mathrm{NL}} = \tilde{u}_x(z,s,n)\sin\beta - \tilde{u}_y(z,s,n)\cos\beta \end{aligned} \tag{8}$$

where the right superscripts "L" and "NL" indicate the linear and nonlinear parts, respectively.

Out of mid-line displacements can also be separate into the linear and non-linear components, i.e.

$$w^{\mathrm{L}}(z,s,n) = \bar{w}^{\mathrm{L}} - n\frac{\partial\bar{u}^{\mathrm{L}}}{\partial z}; \quad v^{\mathrm{L}}(z,s,n) = \bar{v}^{\mathrm{L}} - n\frac{\partial\bar{u}^{\mathrm{L}}}{\partial s}; \quad u^{\mathrm{L}}(z,s,n) = \bar{u}^{\mathrm{L}} \tag{9}$$

$$w^{\mathrm{NL}}(z,s,n) = \bar{w}^{\mathrm{NL}} - n\frac{\partial\bar{u}^{\mathrm{NL}}}{\partial z}; \; v^{\mathrm{NL}}(z,s,n) = \bar{v}^{\mathrm{NL}} - n\frac{\partial\bar{u}^{\mathrm{NL}}}{\partial s}; \; u^{\mathrm{NL}}(z,s,n) = \bar{u}^{\mathrm{NL}} \tag{10}$$

and according to the geometrical hypothesis of the in-plane rigidity for the cross-section, the only non-zero strain increments are:

$$e_{zz} = \frac{\partial w^{\mathrm{L}}}{\partial z} \quad e_{zs} = \frac{\partial w^{\mathrm{L}}}{\partial s} + \frac{\partial v^{\mathrm{L}}}{\partial z}; \tag{11}$$

$$\begin{aligned} & \eta_{zz} = \frac{1}{2}\left[\left(\frac{\partial w^{\mathrm{L}}}{\partial z}\right)^2 + \left(\frac{\partial u^{\mathrm{L}}}{\partial z}\right)^2 + \left(\frac{\partial v^{\mathrm{L}}}{\partial z}\right)^2\right]; \\ & \eta_{zs} = \frac{\partial w^{\mathrm{L}}}{\partial z}\frac{\partial w^{\mathrm{L}}}{\partial s} + \frac{\partial u^{\mathrm{L}}}{\partial z}\frac{\partial u^{\mathrm{L}}}{\partial s} + \frac{\partial v^{\mathrm{L}}}{\partial z}\frac{\partial v^{\mathrm{L}}}{\partial s}; \end{aligned} \tag{12}$$

$$\tilde{e}_{zz} = \frac{\partial w^{\mathrm{NL}}}{\partial z}\;; \quad \tilde{e}_{zs} = \frac{\partial w^{\mathrm{NL}}}{\partial s} + \frac{\partial v^{\mathrm{NL}}}{\partial z} \tag{13}$$

where, respectively, $e_{ij}$ and $\eta_{ij}$ are the linear and non-linear strains due to the first-order displacement increments given by Eq. (2),