Composite Structures 134 (2015) 247–254

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

A tensegrity approach to the optimal reinforcement of masonry domes and vaults through fiber-reinforced composite materials

F. Fraternali [a,*], G. Carpentieri [a,b], M. Modano [b], F. Fabbrocino [c], R.E. Skelton [d]

[a] Department of Civil Engineering, University of Salerno, 84084 Fisciano, SA, Italy
[b] Department of Structural Engineering, University of Naples Federico II, 80132 Naples, Italy
[c] Department of Engineering, Pegaso University, Piazza Trieste e Trento, 48, 80132 Naples, Italy
[d] Department of Mechanical and Aerospace Engineering, University of California San Diego, La Jolla, CA, 92093-0411, USA

ARTICLE INFO

Article history:
Available online 28 August 2015

Keywords:
Masonry
Tensegrity
Topology optimization
Minimal mass
Vaults
Domes

ABSTRACT

We present a tensegrity approach to the strengthening of masonry vaults and domes performed by bonding grids of fiber reinforced composites to the masonry substrate. A topology optimization of such a reinforcement technique is formulated, on accounting for a tensegrity model of the reinforced structure; a minimal mass design strategy; different yield strengths of the masonry struts and tensile composite reinforcements; and multiple loading conditions. We show that the given optimization strategy can be profitably employed to rationally design fiber-reinforced composite material reinforcements of existing or new masonry vaults and domes, making use of the safe theorem of limit analysis. A wide collection of numerical examples dealing with real-life masonry domes and vaults highlight the technical potential of the proposed approach.

1. Introduction

The field of Discrete Element Modeling (DEM) of materials and structures is growing rapidly, attracting increasing attention from physicists and mechanicians working in different research areas. Originally, such a computational technique was aimed at describing particle interactions in discrete systems, via suitable force and/or torque systems (fully discrete systems, refer, e.g., to [1] and references therein). Nowadays, DEMs are also frequently used in association with continuous approximation schemes (coupled discrete-continuum models), in order to tackle scaling limitations of purely discrete models. DEMs may indeed require a large number of variables, being well suited to describe small process zones (dislocation and fracture nucleation, nanoindentation, atomic rearrangements, etc., cf. [2–7]).

In structural mechanics, a special class of DEMs is that of equivalent truss models of solids and structures, which includes Lumped Strain/Stress Models (LSM) of plates and shells [8–10]; Thrust Network Approaches (TNA) to masonry structures [11–16]; mechanical models of chains of granular materials or carbon nanotube (CNT) arrays [17,18]; and strut and tie models of discontinuous regions in reinforced-concrete structures [19], just to name a few examples. Some convergence studies of such methods in the continuum limit are presented in [20–22] for bending plates, 2D elasticity, and CNT arrays, respectively.

Tensegrity structures are prestressable truss structures, which are obtained by stabilizing a set of compressed members (bars or struts) through a network of tensile elements (cables or strings). Tensegrity architectures have been used to describe a large variety of natural [23] and engineering systems [24–26], and it has been shown that the tensegrity approach to structural mechanics leads to design minimal mass systems in different mechanical problems [27–32].

The present work deals with the topology optimization of reinforcements of masonry vaults and domes realized through meshes of Fiber Reinforced Polymers (FRP) and/or Fabric Reinforced Cementitious Matrix (FRCM) composites bonded to the masonry substrate. We model the examined structures as tensegrity networks of masonry struts and tensile elements corresponding to the FRP-/FRCM-reinforcements. Such reinforcements are often applied to masonry structures in the form of meshes of 1D elements [33,34], and are aimed at carrying tensile forces that would otherwise cause cracking damage of masonry [35–39]. The proposed optimization strategy determines the minimal mass tensegrity structure connecting a given node set, under prescribed yielding constraints. Each node is potentially connected to all the

* Corresponding author.
E-mail addresses: f.fraternali@unisa.it (F. Fraternali), gcarpentieri@unisa.it (G. Carpentieri), modano@unina.it (M. Modano), francesco.fabbrocino@unipegaso.it (F. Fabbrocino), bobskelton@ucsd.edu (R.E. Skelton).

http://dx.doi.org/10.1016/j.compstruct.2015.08.087
0263-8223/

neighbor nodes lying in a ball of given radius, through compressive and tensile elements. Such a connection pattern defines a background structure that is subject to minimal mass optimization [30], assuming different yield strengths for the masonry struts (compressive elements), and the FRP/FRCM reinforcements. An optimization procedure takes the node set defining the geometry of the structure (obtained, e.g., through a laser-scanner), the material density and the compressive and tensile material strengths as input parameters. It produces a minimal mass resisting mechanism of the reinforced structure as output, which can be regarded as a lumped stress/thrust network/strut and tie model of the examined structure [12,14,15]. Under the assumption of perfectly plastic response of masonry in compression and FRP/FRCM reinforcements in tension, the safe theorem of the limit analysis of elastic–plastic bodies [40] ensures that the reinforced structure is safe under the examined loading conditions. It is worth noting that the *Italian Guide for the Design and Construction of Externally Bonded FRP Systems for Strengthening Existing Structures* claims what follows: 'Simplified schemes can also be used to describe the behavior of the structure. For example, provided that tensile stresses are directly taken by the FRP system, the stress level may be determined by adopting a simplified distribution of stresses that satisfies the equilibrium conditions but not necessarily the strain compatibility' (see [39], Section 5.2.1). A minimal mass resisting mechanism allows for an optimized design of the FRP-/FRCM-reinforcements, preventing excessive over-strength of the reinforced structure, which may be responsible for reduced 'cracking-adaptation' capacity [41].

It worth noting that the strengthening of pre-existing masonry structures may require the application of a suitable state of prestress to be effective [39].

The paper is structured as follow. Section 2 describes the proposed tensegrity model of a reinforced masonry vault or dome, which is based on an automatically generated background structure. Next, Section 3 formulates a minimum mass optimization of such a structure, under given yielding constraints and multiple loading conditions. The following Section 4 presents a parade of case studies dealing with FRP-/FRCM- reinforcements of a dome (Section 4.1), a groin vault (or cross vault), a cloister vault (or domical vault) and a barrel vault (Section 4.2). Concluding remarks and prospective work are illustrated in Section 5.

2. Tensegrity model of a reinforced masonry vault

Let us consider a masonry vault or dome with mean surface described by a set of n_n nodes in the 3D Euclidean space. In a given Cartesian frame $\{O, x, y, z\}$, the components (x_k, y_k, z_k) of the position vectors $\mathbf{n}_k$ of all such nodes ($k = 1, \ldots, n_n$) can be arranged into the following $3 \times n_n$ *node matrix*

$$\mathbf{N} = \begin{bmatrix} x_1 & \ldots & x_{n_n} \\ y_1 & \ldots & y_{n_n} \\ z_1 & \ldots & z_{n_n} \end{bmatrix} \tag{1}$$

We now introduce a *background structure*, which is obtained by connecting each node $\mathbf{n}_k$ with all the neighbors $\mathbf{n}_j$ such that it results $|\mathbf{n}_k - \mathbf{n}_j| \leqslant r_k$ (*interacting neighbors*). Here, $|\mathbf{n}_k - \mathbf{n}_j|$ is the Euclidean distance between $\mathbf{n}_k$ and $\mathbf{n}_j$, and r_k is a given *connection radius*. Fig. 1 shows the particular case in which the interacting neighbors of a selected node coincide with its nearest neighbors. We connect $\mathbf{n}_k$ to each interacting neighbor $\mathbf{n}_j$ through two elements working in parallel: a compressive masonry strut (or *bar*) $\mathbf{b}_i = \mathbf{n}_k - \mathbf{n}_j$, and a tensile FRP/FRCM element (or *string*) $\mathbf{s}_i = \mathbf{n}_k - \mathbf{n}_j$. The minimal mass optimization of the background structure presented in Section 3 will choose which one such members (bar or string) is eventually present between nodes $\mathbf{n}_k$ and $\mathbf{n}_j$ in the optimized configuration (i.e., which one of the above members eventually carries a nonzero axial force in the minimal mass configuration, see also [30], Section 7). For future use, we let n_b and n_s denote the total number of bars and the total number of strings composing the background structure, respectively (with $n_b = n_s$ in the non-optimal configuration), and we set $n_x = n_b + n_s$.

We assume that the background structure is subject to a number m of different loading conditions, and, with reference to the j-th condition, we let $\lambda_{b_i}^{(j)}$ denote the compressive force per unit length (force density) acting in the i-th bar, and let $\gamma_{s_i}^{(j)}$ denote the tensile force per unit length acting in the i-th string, both defined to be positive quantities. The static equilibrium equations of the nodes in correspondence of the current load condition can be written as follows

$$\boldsymbol{A}\boldsymbol{x}^{(j)} = \boldsymbol{w}^{(j)} \tag{2}$$

where $\boldsymbol{A}$ is the $3n_n \times n_x$ *static matrix* of the structure, depending on the geometry and the connectivity of bars and strings (see [30]); $\boldsymbol{w}^{(j)}$ is *external load vector*, which stacks the $3n_n$ Cartesian components of the external forces acting on all nodes in the current loading condition; and $\boldsymbol{x}^{(j)}$ is the vector with n_x entries that collects the force densities in bars and strings in correspondence of the same loading condition, that is

$$\boldsymbol{x}^{(j)} = [\lambda_1^{(j)} \cdots \lambda_{n_b}^{(j)} | \gamma_1^{(j)} \cdots \gamma_{n_s}^{(j)}]^T \tag{3}$$

Let σ_{b_i} and σ_{s_i} respectively denote the compressive strength of the generic bar and the tensile strength of the generic string forming the background structure, which we hereafter assume behaving as perfectly plastic members. Yielding constraints in bars and strings require that, for each loading condition, it results

$$\lambda_i^{(j)}\ b_i \leqslant \sigma_{b_i} A_{b_i}, \quad \gamma_i^{(j)}\ s_i \leqslant \sigma_{s_i} A_{s_i} \tag{5}$$

where A_{bi} and A_{si} respectively denote the cross-section areas of the generic bar and string.

The masses of the generic bar and string of the background structure are computed as follows

$$m_{b_i} = \rho_{b_i} A_{b_i} b_i, \quad m_{s_i} = \rho_{s_i} A_{s_i} s_i, \tag{6}$$

where ρ_{b_i} and ρ_{s_i} denote the mass densities of such members, respectively.

3. Minimal mass design

Following [30], we formulate a *minimal mass design* of the background structure through the following linear program

$$\begin{aligned} &\underset{\boldsymbol{x}^{(j)}, \boldsymbol{y}}{\text{minimize}} \qquad m = \boldsymbol{d}^T \boldsymbol{y} \\ &\text{subject to} \qquad \begin{cases} \boldsymbol{A}\boldsymbol{x}^{(j)} = \boldsymbol{w}^{(j)} \\ \boldsymbol{C}\boldsymbol{x}^{(j)} \leqslant \boldsymbol{D}\boldsymbol{y} \\ \boldsymbol{x}^{(j)} \geqslant \boldsymbol{0}, \boldsymbol{y} \geqslant \boldsymbol{0} \end{cases}, \end{aligned} \tag{7}$$

where

$$\boldsymbol{y} = [A_{b_1} \cdots A_{b_{n_b}} | A_{s_1} \cdots A_{s_{n_s}}]^T \tag{8}$$

$$\boldsymbol{d}^T = [\varrho_{b_i} b_i \cdots \varrho_{b_{n_b}} b_{n_b} | \varrho_{s_i} s_i \cdots \varrho_{s_{n_s}} s_{n_s}] \tag{9}$$

$$\boldsymbol{C} = \begin{bmatrix} \text{diag}(b_1, \cdots, b_{n_b}) & \boldsymbol{0} \\ \boldsymbol{0} & \text{diag}(s_1, \cdots, s_{n_s}) \end{bmatrix} \tag{10}$$

$$\boldsymbol{D} = \begin{bmatrix} \text{diag}(\sigma_{b_1}, \cdots, \sigma_{b_{n_b}}) & \boldsymbol{0} \\ \boldsymbol{0} & \text{diag}(\sigma_{s_1}, \cdots, \sigma_{s_{n_s}}) \end{bmatrix} \tag{11}$$