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Fatigue crack growth in functionally graded material using homogenized XIGA

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ABSTRACT

In this work, the effect of defects/flaws (holes, inclusions, cracks) on the fatigue life of functionally graded material (FGM) is analyzed by homogenized extended isogeometric analysis (XIGA). In FGM, the gradation in the material properties is taken along the length of the plate. In XIGA, the crack faces are modeled by discontinuous Heaviside jump function, whereas the singularity in stress field at the crack tip is modeled by crack tip enrichment functions. Holes and inclusions are modeled by Heaviside jump function and distance function, respectively. The values of stress intensity factor (SIF) are numerically evaluated using the domain form of interaction integral approach. Paris law of fatigue crack growth is employed for computing the fatigue life. The flaws are modeled in a 30% region near the main crack, while the rest of the region is modeled with an equivalent homogeneous material. Several problems involving discontinuities in 30% region of the domain are solved by XIGA, and the results are compared with those obtained by modeling discontinuities in the entire domain of the plate.

1. Introduction

Composite materials are made of two or more constituents, which are distinct, separate at the microscopic or macroscopic level, and have significantly different physical and chemical properties. In general, any material consisting of two or more constituents with different properties and distinct boundaries among the components are referred as composite materials. Functionally graded material (FGM) is also a type of the composite materials in which the composition and micro-structure of its constituents vary as per need. FGMs are characterized by local variation in their composition/microstructure or both to obtain a useful variation in the local material properties. The different micro-structural phases in FGMs exhibit different properties so that the overall FGMs achieve a multi-structural status from the properties gradation point of view. The microstructure of FGMs is generally heterogeneous and mainly failure occurs due to crack nucleation. The presence of flaws such as holes and inclusions in these material complicates the modeling. Therefore, from the computational cost point of view, often microscopic models are used to evaluate the overall characteristics of the heterogeneous material [1].

The microscopic analysis of the structures/components containing static and growing cracks becomes quite important under fatigue loading. Several effective numerical models such as homogenization [63,18,1] and multi-scale [64,43,41] are employed. Budarapu et al. [17] proposed a multi-scale method (atomistic continuum numerical method) for the analysis of quasi-static crack growth. Talebi et al. [62] proposed a method to couple a three-dimensional continuum domain with molecular dynamic domain to simulate propagating cracks.

Till date, several homogenization approaches have been proposed such as Mori–Tanaka [47], self-consistent [22] and differential scheme [44], asymptotic method [56], multipole expansion [40] approach based on fast Fourier transform [48,45,16] and strain energy based homogenization approach [1]. Among these, the strain energy based homogenization approach is found quite effective from the computational cost point of view. In this approach, the heterogeneous material of the structure is replaced by an equivalent homogeneous material. According to this approach, the strain energy density of the actual heterogeneous RVE must be equal to the strain energy density of the equivalent homogeneous RVE under same loading and boundary conditions. To evaluate the equivalent homogeneous material properties of FGM, a two-step procedure is adopted. Firstly, FGM is treated as an equivalent composite and the properties are evaluated using rule of mixtures. Further, the equivalent material properties of equivalent composite are evaluated in the presence of flaws using

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homogenization approach. The main hypothesis in this methodology is that all statistical averaged properties of the state variables are same at any point in the material.

In past, some efforts have been made to study and investigate the behavior of FGMs. Prabhakar and Tippur [51] conducted various experiments for the computation of stress fields at the crack tip. Kim and Paulino [36] employed finite element method for the computation of mixed mode SIF in FGM. Dolbow and Gosz [27] and Rao and Rahman [54] employed interaction energy contour integrals for the evaluation of mixed mode SIF in 2-D orthotropic FGM. Bahr et al. [3] analyzed the crack growth in FGM using weight function method. Zhang et al. [67] used boundary integral equation method for the mixed mode crack growth analysis of uni-directional and bi-directional FGMs. Huang et al. [33] used the multi-layered model for FGM. Zhang et al. [68] performed the elasto-static analysis of anti-plane cracks in unidirectional and bi-directional FGMs using hyper-singular boundary integral equation method. Carpinteri et al. [20] performed brittle crack propagation and fatigue crack growth in FGM using finite element method. Carpinteri and Pugno [19] computed the strength of FGM structures by introducing the re-entrant corners. Chakraborty and Rahman [21] presented three multi-scale models for the fracture analysis of a crack in two phase functionally graded composites. Guo and Noda [30] analytically investigated functionally graded layered structure with a crack crossing the interface. Comi and Mariani [23] addressed fracture processes in quasi brittle FGM with ad-hoc extended finite element method. Dag et al. [25] investigated the mixed mode periodic crack problems in orthotropic FGM using enriched FEM. Bhattacharya et al. [13] evaluated the fatigue life of FGM using extended finite element method under mode-I loading.

To accurately simulate the behavior of cracked structures, a number of numerical methods such as element free Galerkin method [5,50], boundary element method [66,65], reproducing kernel particle method [42], meshless local Petrov Galerkin method [2], coupled FE-EFGM [37,59], cracking particle method [52,53] and extended finite element method [4,37]. In all these methods, the approximation of geometry introduces some error in the solution as different basis functions are employed for defining the geometry and solution. To cope-up with this, Hughes et al. [34] introduced a powerful numerical tool, known as isogeometric analysis (IGA). In IGA, the error associated with the domain discretization is totally removed as the same basis functions are employed for defining the geometry as well as the solution. Recently, the IGA was extended to tackle the problems involving defects using PU enrichment, and was named as extended isogeometric analysis (XIGA). Benson et al. [6] analyzed the fracture mechanics problems using XIGA. Haasemann et al. [31] employed XIGA to inspect a bi-material body with curved interfaces. De Luycker et al. [26] established that XIGA provides greater accuracy and higher convergence rate to solve the linear elastic fracture mechanics problems. Ghorashi et al. [29] used XIGA to perform the fracture analysis of structures. Jia et al. [35] extended the IGA to solve material interface problems. Ghorashi et al. [28] used T-spline based XIGA for the fracture analysis of orthotropic media. Nguyen-Thanh et al. [49] used XIGA for the analysis of through-thickness cracks in thin shell structures. Bhardwaj and Singh [8] carried out the fatigue crack growth analysis in homogenous material in the presence of flaws using XIGA. They [9,10] solved few crack problems in the homogeneous and functionally graded cracked plate using XIGA under different loading and boundary conditions.

In the present work, XIGA has been extended for the simulation of crack/crack growth in FGMs. A parametric study is conducted by Bhardwaj et al. [11] to decide a region of discontinuities or micro-defects which have major influence on the fatigue life. They found that the micro-defects present in 30% region of the domain near the major crack, largely influence the fatigue life of the plate. Thus, in this work, the defects are modeled only in 30% region of the domain while the remaining domain is modeled with the properties of equivalent homogenous material. The main objectives of present study are as follows:

- To simulate cracks in functionally graded materials using XIGA.
- To perform the fatigue crack growth analysis of an edge cracked FGM plate in the presence of flaws.
- To reduce the computational time by the use of homogenization for the analysis of cracked FGM plate.

This paper is organized as follows: the frame work of isogeometric analysis (basis function, knot vector and isogeometric discretization) is discussed in Section 2. Section 3 describes the formulation of extended isogeometric analysis, including the approximations for cracks, holes, inclusions, interfaces. Section 4 depicts a methodology for SIFs computation, criterion for fatigue crack growth in FGMs and the physics of FGM along with its relevant properties. The homogenization process and evaluation of equivalent material properties are described in Section 5. Several numerical problems are illustrated in Section 6 for evaluating the fatigue life of the edge cracked FGM. The conclusions are presented in Section 7.

2. Isogeometric analysis

Non-uniform rational B-splines (NURBS) are commonly used in computer aided design due to their ability to represent complex geometries exactly. Isogeometric analysis (IGA) uses identical basis functions for defining the geometry and solution. In IGA, the error due to geometric discretization is totally removed as same basis functions (NURBS) are used for defining the geometry and the solution. The details of the basis function, knot vector and isogeometric discretization are given below.

2.1. Basis function

In this section, B-splines, NURBS and the derivatives of NURBS basis functions are briefly presented. B-splines are made from the piecewise polynomial functions. For a detailed description one can refer to Cottrell et al. [24]. Consider a one-dimensional parametric space $\xi \in [0,1]$; a knot vector Ξ to construct the B-spline basis functions, is a set of non-decreasing real numbers, i.e. $\Xi = \{\xi_1 = 0, \xi_2, \dots \xi_{n+p+1} = 1\}$ with $\xi_i \in R$ and $\xi_i \leqslant \xi_{i+1}$, where ξ_i, n and p represent a knot, the number of basis functions and the order of basis functions, respectively. The non-zero interval $[\xi_i, \xi_{i+1}]$ (termed as knot span), is known as element in IGA. An open knot vector (two ends of knots are repeated $p+1$ times) is given as,

$$\Xi = \{\xi_1 = \dots \xi_{p+1} = 0, \xi_{p+2}, \dots, \xi_n, \xi_{n+1} = \dots \xi_{n+p+1} = 1\} \tag{1}$$

The B-spline basis functions of degree p, $N_{i,p}(\xi)$ are defined using the Cox–de Boor recursion formula,

$$N_{i,0}(\xi) = \begin{cases} 1 & \xi_i \leqslant \xi \leqslant \xi_{i+1} \\ 0 & \text{otherwise} \end{cases} \quad \text{for } p = 0 \tag{2}$$

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi) \quad \text{for } p \geqslant 1 \tag{3}$$

The first derivatives of B-spline basis functions required for the computation of element stiffness matrix can be determined for a polynomial degree p and knot vector Ξ as,