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# Magnetoelectric analysis of a bilayer piezoelectric/magnetostrictive composite system with interfacial effect

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ABSTRACT

A magnetoelectric (ME) coupling model to analyze the influence of the interfacial properties on the ME behaviors of a bilayer composite system which consisting magnetostrictive (MS) and piezoelectric (PE) layers and works in bend mode are investigated in this article, and the closed form analytical solution is presented. The interface of the bilayer system between MS and PE layer is assumed to be imperfectly connected and is modeled by the shear-lag model. A sixth-order differential equation governing the displacement is derived and its analytical solution is derived. The effect of interfacial property on the static and dynamic ME behaviors includes the average output electrical power density of vibration-based ME bilayer system is discussed. The present analytic results can be degenerated to the ones for a bilayer system with perfect interfaces. It can be found that the interfacial properties play a critical role in the performance characteristics of the ME bilayer system. A quite different feature from previous work is that the static ME effect founded to be length size-dependent. The potential applications of this theoretical analysis can be found in evaluating the performance of the MS–PE composite with imperfectly interface.

## 1. Introduction

Magnetoelectric (ME) materials are a special class of materials which have the ME effect which is described as the induced polarization in an applied magnetic field, or an induced magnetization in an applied electric field [1]. The ME materials have drawn increasing interest for this special characteristic which can enhance functionality in sensors and other devices. The development of ME materials has experienced from a single phase compound to particulate composites, and finally layer composites [2–4]. The ME effect in most of the single phase compound is largely limited for practical devices due to their weak ME effect and low critical temperatures. Recently, the remarkably larger ME effect has been found in layered composites. In this type structure, the electric and magnetic field coupled together through interfacial mechanical coupling between the magnetostrictive (MS) and piezoelectric (PE) layers. Strong ME effect has been demonstrated at room temperature which exhibited great potential in practical devices applications.

In the past few years, numerous theoretical studies have been carried out to investigate the ME effect in ME composites. Ryu et al. have present an analytical model for layered disk-type plate [5]. Dong et al. have developed an equivalent circuit approach for the electromechanical resonant (EMR) analysis on layered beam-type composites [6]. Other type ME composite configure structures, like ring-disk which working in radial mode are also investigated [7]. However, layered beam-type structures are the most common configurations in engineering and microelectromechanical system (MEMS) applications. In most previous works, it is assumed that the interface between the PE layer and MS layer is perfectly connected. However, it is not always the case in practical conditions. The interface between two different materials may be weakened in the processing of manufacturing as a results of the involvement of small micro cracks and inhomogeneities or may be damaged during the time service under various load conditions. Thus, authors have proposed a model that includes interface coupling factor to account for weak interface connection [8]. While this provides an analysis supported to the experimental results, it is unlikely that interface slip occurs at the well-bonded interface. To analyze the interfacial effect, the interface slip has studied as shear-lag effect to predict load transforms between piezoelectric actuators and an elastic substrate. Based on the Euler–Bernoulli theory and shear-lag model, vibrations of piezoelectric-substrate bilayer beams with interface slip were investigated in Refs. [9,10]. Although the shear-lag analysis is widely used in the passive

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piezoelectric composite systems, this approach has not been widely developed for active composite systems, like ME composites structure. Modeling static shear-lag and demagnetization effects in ME symmetry layer system (MS–PE–MS) was proposed by Chang [11], and dynamic response model of this structure was developed by Hasanyan [12]. However, these modeling approaches were applicable only on to the symmetry structures (i.e. one PE layer sandwiched by two MS layers or one MS layer sandwiched by two PE layers). For non-symmetrical configuration, the structure implies flexural strain which induced the field functions vary not only along the longitudinal direction but also through the thickness as well. Thus, the influences of flexural strain on ME behaviors in non-symmetrical structure must be considered. Furthermore, the results obtained in resonance modes including the longitudinal mode in symmetry layered structures and radial mode in ring-disk structures indicated that the operating frequency are generally high, which could bring significant eddy current loss for the MS phase, and finally resulting for an inefficient energy convert [13]. Thus, a non-symmetrical configuration with MS–PE bilayer ME composite that operates in bend-resonance mode need to deep investigation due to its simple structure and low-resonance frequency characteristics.

To capture the effect of the interfacial properties on the ME effect of the ME bilayer composites system, an analytic model is always required. This paper is intended to present the static and dynamic ME behaviors analysis of ME bilayer composites with imperfectly interface based on the Euler–Bernoulli beam theory and shear-lag model. A sixth-order differential equation governing the displacement of the beam is thereby derived. The exact solution of the equation is presented and the ME effect in the bilayer system is obtained for illustration. Furthermore, the degenerated models for static and perfectly interface are also investigated. Using these equations, a numerical analysis is performed for this bilayer structure.

## 2. Governing equations

Considering a MS–PE bilayer composite of length $L$, width $B$, and total thickness $2h$, as shown in Fig. 1. The composite consists of a PE layer and a MS layer. The thickness of the PE layer and MS layer is defined as $2h_p$ and $2h_m$, respectively. Here the distance between the centroids of the PE layer and the MS layer is defined as $h = (h_p + h_m)$. The PE layer is polarized in the negative $z$-axis and covered by very thin electrodes on its top and bottom surfaces. The electrode coated on the lower surface of the PE layer is grounded. Static (DC bias magnetic) and alternating (AC magnetic) magnetic fields were applied along the longitudinal direction and across the plane of the contacts. Due to magnetostriction, a magnetic field induced force oscillation of the MS layer, and then is transfer to PE layer, finally generates a voltage in the PE layer through the piezoelectric effect. Owing to the asymmetric characteristics of the bilayer system, non-uniformly distributed (along thickness) stress causes extensional deformation as well as the flexural deformation. The extensional deformation dominates in longitudinal vibration resonance mode, whereas the flexural deformation dominates in bending vibration resonance mode. ME behaviors of the longitudinal vibration resonance mode can be predicted in Ref. [12]. Here, we will focusing on analysis the bending vibration resonance mode.

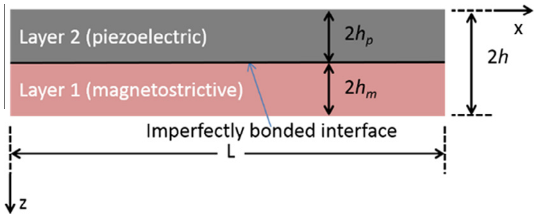

**Fig. 1.** Schematic diagram of the ME bilayer composite system with imperfectly interface.

In the Cartesian coordinate system $(x, z)$, under the one-dimensional stress assumption in beam theory, the constitutive equation describing the linear coupling between stress, strain and the fields is given below. Here, the version with stress and fields as independent variables is used, i.e., Eqs. (1) and (2) for the MS layer, and, (3) and (4) for the PE layer [14].

$$\varepsilon_{3m} = s^H_{33m}\sigma_{3m} + d_{33m}H_3 \tag{1}$$

$$B_3 = d_{33m}\sigma_{3m} + \mu^T_{33}H_3 \tag{2}$$

$$\varepsilon_{1p} = s^D_{11}\sigma_{1p} + g_{31p}D_3 \tag{3}$$

$$E_3 = -g_{31p}\sigma_{1p} + \beta^T_{33}D_3 \tag{4}$$

Here, $\varepsilon$ and $\sigma$ denotes the strain and stress and $D$, $B$, $H$ and $E$ the electric displacement, magnetic induction, magnetic electric fields, respectively. The elastic compliances are given by $s$ with subscripts $m$ and $p$ denoting the MS and PE layer, respectively. Please note that the material properties tensor elements are given in the local coordinate system (axes denoted 1–3). It is different between the MS and PE layers, i.e., for MS layer $x$ corresponding to 3, and for PE corresponding to 1. Furthermore, $d_{33m}$ denotes the piezomagnetic coefficient, $g_{31p}$ the transverse piezoelectric constant, and $\beta^T_{33}$ the inverse dielectric constant, respectively.

Suppose the interface between the MS and PE layer is not perfectly contact. Define $u_s$ as the interlayer displacement slip and $T_s$ the interlaminar shear force, respectively. By introducing an interfacial parameter $K_s$, the shear-lag model is then written as [9]

$$u_s = K_sT_s \tag{5}$$

The larger $K_s$, the weaker connection at the interface. Consider two limiting cases: (I) $K_s = 0$ corresponding to $u_s = 0$, which means that the interface is perfectly contact, and (II) $K_s \to \infty$ corresponds to $T_s = 0$, which means that the interface is smooth contact and the two layers are uncoupled.

To study the weak interface effect, the following assumptions are adopted: (1) all the constitutive materials are linear and the deformation are small; (2) Euler–Bernoulli theory is not applicable for the whole cross-section, but is still valid for each individual layers; (3) no transverse separation occurs on the connection interface, which means that the curvature is the same for both sub-layer at any cross-section; (4) the section plane remain plane after deformation in each individual layer. Based on these assumptions, the displacement fields are assumed to be

$$w_p(x,z,t) = w_m(x,z,t) = w(x,t) \tag{6}$$

$$u_p(x,z,t) = u_{p0}(x,t) - z_p\frac{\partial w(x,t)}{\partial x}, \quad (-h_p \leqslant z_p \leqslant h_p) \tag{7}$$

$$u_m(x,z,t) = u_{m0}(x,t) - z_m\frac{\partial w(x,t)}{\partial x}, \quad (-h_m \leqslant z_m \leqslant h_m) \tag{8}$$

where $w_p(x,z,t)$ and $w_m(x,z,t)$ are the transverse deflection of the PE layer and MS layer, respectively; $u_{p0}(x,t)$ and $u_{m0}(x,t)$ are the axial displacement at the centroids of the PE and MS layers, respectively; $u_p(x,z,t)$ and $u_m(x,z,t)$ are the axial displacement components in the PE and MS layer; and $z_p$ and $z_m$ denotes the local thickness coordinate with the origin located on the middle plane in the PE and MS layer, respectively.