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Bending analysis of FG microbeams resting on Winkler elastic foundation via strain gradient elasticity

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ABSTRACT

In this study, bending response of non-homogenous microbeams embedded in an elastic medium is investigated based on modified strain gradient elasticity theory in conjunctions with various beam theories. The governing differential equations and related boundary conditions are derived with the aid of minimum total potential energy principle. Elastic medium is modeled by Winkler foundation model. Bending problem of simply supported microbeams made of functionally graded materials is solved by Navier's solution procedure. A detailed parametric study is performed to investigate the effects of the material length scale parameters-to-thickness ratio *l/h*, material property gradient index *k*, slenderness ratio *L/h*, Winkler modulus *kw* and shear correction factor on the bending behavior of embedded functionally graded microbeams.

1. Introduction

In traditional laminated composites, the interconnected each layer has different material properties. Unfortunately, because of sudden change in material properties, high shear stress problem may occur at the interface of two adjacent layers. Unlike in laminated composites, material properties vary continuously and smoothly throughout the certain dimension(s) in functionally graded materials (FGMs) that can be defined as a relatively new improved kind of composite materials. It can be emphasized that FGMs may be a good solution for undesirable stress concentrations. Consequently, structures made of FGMs have a wide range of applications in many industries such as aerospace, biomedicine, mechanical, nuclear, electronics and optics due to their novel thermo-mechanical properties. There are a number of studies in the literature on investigation mechanical characteristics of functionally graded (FG) structures with various solution methods [1–9].

Microbeams are one of the most commonly used structures in micro- and nano-electro mechanical systems (MEMS and NEMS) such as micro-resonators [10], Atomic Force Microscopes [11], micro-switches [12] and micro-actuators [13]. Some experimental observations showed that size effect plays an important role on mechanical responses of small-sized structures [14–16]. For instance, it is observed by Lam et al. [16] in micro bending test of epoxy beams that the normalized bending rigidity increases about 2.4 times as the thickness of the microbeam reduces from 115 to 20 μm. Classical continuum mechanics is not sufficient to predict the small scale effect of micro- and nano-sized structures in the absence of any material length scale parameters. Hence, several non-classical continuum theories in which there is at least one additional material length scale parameter have been developed to determine the mechanical characteristics of small-sized structures, such as couple stress theory [17–19], micropolar theory [20], nonlocal elasticity theory [21,22] and strain gradient theories [16,23–26].

The modified strain gradient theory (MSGT) was introduced by Lam et al. [16] in which strain energy density includes dilatation gradient vector, deviatoric stretch and symmetric rotation gradient tensors besides symmetric strain tensor. For linear elastic isotropic materials, the formulations and governing equations contain three additional material length scale parameters relevant to higher-order deformation gradients besides two classical ones. Many studies have been performed to investigate mechanical responses of homogeneous microbars [27–30] and microbeams [31–36].

Another type of higher-order continuum theories is modified couple stress theory (MCST) proposed by Yang et al. [37]. Unlike MSGT, this theory includes only one material length scale parameter related to symmetric rotation gradient tensor and has been employed to analyze mechanical responses of microbeams [38–45].

Nowadays, FG microstructures are extensively used in MEMS and NEMS [46–48] due to the rapid advances in nanotechnology.

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Several studies have been performed to determine mechanical characteristics of FG microbars and microbeams. For instance, Akgöz and Civalek [49] investigated longitudinal free vibration analysis of axially functionally graded (AFG) microbars based on MSGT in conjunctions with Rayleigh–Ritz solution method. Sadeghi et al. [50] presented a study about the effect of material length scale parameters on analysis of strain gradient FG micro-cylinders. Static bending and free vibration analysis of simply supported FG microbeams based on Bernoulli–Euler beam model and MSGT was investigated by Kahrobaiyan et al. [51]. Also, buckling analysis of strain gradient FG microbeams for different boundary conditions on the basis of Bernoulli–Euler beam theory was performed by Akgöz and Civalek [52]. A size-dependent strain gradient Timoshenko beam model for nonhomogenous microbeams was presented by Ansari et al. [53].

In addition to the aforementioned studies based on Bernoulli–Euler (EBT) and Timoshenko (TBT) beam theories, there are several works on investigation mechanical responses of such structures on the basis of higher-order shear deformation beam theories including parabolic (third-order) beam theory (PBT) [54,55], trigonometric (sinusoidal) beam theory (SBT) [56], hyperbolic beam theory [57], exponential beam theory [58] and a general exponential beam theory [59] in conjunctions with modified couple stress and strain gradient theories [60–72].

In this work, a microstructure-dependent trigonometric beam model is presented to investigate the static bending behavior of embedded FG microbeams. The governing equations and corresponding boundary conditions are obtained by implementing minimum total potential energy principle. Winkler elastic foundation model is used to simulate the interactions between FG microbeam and elastic medium. Bending problem of simply supported embedded FG microbeams is solved by Navier's solution procedure. A detailed parametric study is performed to investigate the effects of the material length scale parameters-to-thickness ratio, gradient index, length-to-thickness ratio, Winkler modulus and shear correction factor on the bending behavior of embedded FG microbeams.

2. Modified strain gradient elasticity theory

One of the non-classical elasticity theories is the modified strain gradient elasticity theory [16], in which, there are higher-order deformation gradients like dilatation gradient vector, deviatoric stretch gradient and symmetric rotation gradient tensors in addition to classical strain tensor. For linear elastic isotropic materials, they can be defined by two Lamé constants and three additional material length scale parameters. The strain energy U can be written with infinitesimal deformations as [16,31]

$$U = \frac{1}{2}\int_0^L \int_A (\sigma_{ij}\varepsilon_{ij} + p_i\gamma_i + \tau_{ijk}^{(1)}\eta_{ijk}^{(1)} + m_{ij}^s\chi_{ij}^s)dAdx \tag{1}$$

where σ_{ij}, p_i, $\tau_{ijk}^{(1)}$ and m_{ij}^s are the components of classical $\boldsymbol{\sigma}$ and higher-order stress tensors $\mathbf{p}$, $\boldsymbol{\tau}^{(1)}$ and $\mathbf{m}^s$ can be described as

$$\sigma_{ij} = \lambda\delta_{ij}\varepsilon_{mm} + 2\mu\varepsilon_{ij} \tag{2}$$

$$p_i = 2\mu l_0^2\gamma_i \tag{3}$$

$$\tau_{ijk}^{(1)} = 2\mu l_1^2\eta_{ijk}^{(1)} \tag{4}$$

$$m_{ij}^s = 2\mu l_2^2\chi_{ij}^s \tag{5}$$

and also the components of the classical strain tensor ε, the dilatation gradient vector γ, the deviatoric stretch gradient tensor $\boldsymbol{\eta}^{(1)}$ and the symmetric rotation gradient tensor $\boldsymbol{\chi}^s$ are respectively represented by ε_{ij}, γ_i, $\eta_{ijk}^{(1)}$ and χ_{ij}^s and are defined as follows

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{6}$$

$$\gamma_i = \varepsilon_{mm,i} \tag{7}$$

$$\eta_{ijk}^{(1)} = \frac{1}{3}(\varepsilon_{jk,i} + \varepsilon_{ki,j} + \varepsilon_{ij,k}) - \frac{1}{15}\left[\delta_{ij}(\varepsilon_{mm,k} + 2\varepsilon_{mk,m}) + \delta_{jk}(\varepsilon_{mm,i} + 2\varepsilon_{mi,m}) + \delta_{ki}(\varepsilon_{mm,j} + 2\varepsilon_{mj,m})\right] \tag{8}$$

$$\chi_{ij}^s = \frac{1}{2}(\theta_{i,j} + \theta_{j,i}) \tag{9}$$

$$\theta_i = \frac{1}{2}e_{ijk}u_{k,j} \tag{10}$$

where u_i represents the components of displacement vector $\mathbf{u}$ and θ_i represents the components of rotation vector $\boldsymbol{\theta}$, also δ and e_{ijk} are the Kronecker delta and permutation symbols, respectively. In addition, l_0, l_1, l_2 are additional material length scale parameters related to dilatation, deviatoric stretch and rotation gradients, respectively. Also, λ and μ are Lamé constants as

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}, \quad \mu = \frac{E}{2(1+\nu)} \tag{11}$$

3. Bending response of embedded FG microbeams

As seen in Fig. 1, the top and bottom surfaces of the embedded FG microbeam are made of pure metal and pure ceramic, respectively and material properties vary along the thickness of the microbeam. The classical rule of mixture is employed to specify the volume fractions of constituents as

$$V_c = \left(\frac{h+2z}{2h}\right)^k, \quad V_m = 1 - \left(\frac{h+2z}{2h}\right)^k \tag{12}$$

where k is the material property gradient index and the subscripts c and m denote the ceramic and metal phase, respectively. According to Eq. (12), the effective material properties (F) can be described as

$$F(z) = (F_c - F_m)V_c + F_m \tag{13}$$

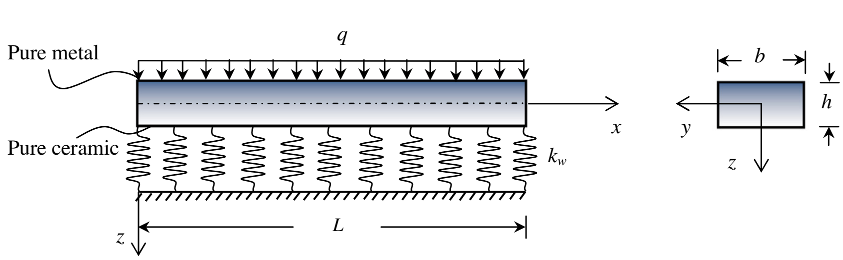

Fig. 1. Geometry and loading of a functionally graded embedded microbeam.