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Computationally efficient reduction of modal data from finite element models by nested sets of B-splines

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ABSTRACT

This paper introduces a novel methodology for the reduction of modal data from complex finite element (FE) models, resulting in a highly accurate and computationally efficient model termed B-SAFE (B-spline-based Semi-Analytical Finite Element model). It is conceptually similar to a method based on nested sets of polynomials proposed earlier by the authors (Poly-SAFE) but provides greater accuracy in the case of complex structures and greater ease of use, limiting the need for user interventions and facilitating automated processing. B-SAFE is constructed via the reduction of a standard displacement-based FE model down to a semi-analytical function built from nested B-splines recursively fitted to modal data, which eliminates the need of high-order polynomials and heavy user-intervention required by Poly-SAFE for the case of complex modal patterns. The present work describes the non-trivial modifications to the original methodology, and shows how fewer and more efficient semi-analytical functions representing field variables such as displacement, strain and stress can be obtained. The new method is exemplified through a numerical case study of a 9.2 m composite wind turbine blade, showing that improved numerical stability and robustness as well as a systematic handling of general geometries can be obtained without user intervention.

1. Introduction

The aeroelastic analysis of flexible structures represents enormous theoretical and computational challenges related to the iterative solution of sizeable, coupled and possibly nonlinear models, in many cases still untreatable with current computational capabilities [1–3]. In view of this, the evaluation of the stress field and the application of failure criteria for layered materials are generally done with simplified models, thus restricting their applicability and adding uncertainty to any design optimization attempt [4–7]. Historically, due to the high cost and scarcity of computing power, the development of reduced-order structural models attracted wide interest [8–12]. Today, despite the exponential growth and decreasing cost of computational resources, the increasingly complex endeavors pursued by researchers and engineers make reduced-order models equally relevant [13–18].

In a recent publication [19], the authors introduced the Poly-SAFE ("Polynomial Semi Analytical Finite Element") method, a reduced-order structural model designed to condense the modal data output of a full-size finite element (FE) model into load-independent semi-analytical functionals via nested polynomials recursively fitted to the modal data. It was shown [19] that Poly-SAFE is capable of retrieving the displacement field at any model coordinate, not restricted to FE nodes, upon simple evaluation of the functional, with an accuracy comparable to that of its parent FE model. Although capable of handling general geometries, the method is particularly suitable for the analysis of Thin-Walled Composite Structures (TWCS) such as those found in large wind turbine blades, slender wings and antennas, among other examples [13–18]. Nonetheless, the use of canonical polynomials (e.g. based on finite power series) in complex modal patterns required either very high-order fits, making them prone to both fitting and numerical instabilities, or a heavy user-involvement to partition the data into piecewise-smooth portions which were fitted individually by low-order polynomials.

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This paper introduces a major improvement to Poly-SAFE for alleviating the aforementioned issues based on the usage of B-splines [20,21], motivating the naming of the new method as B-SAFE ("B-spline-based Semi Analytical Finite Element"). The novel approach allows a similar but more robust representation of semi-analytical functionals as well as an extension of the capabilities of the original method due to the advanced arithmetic/differential manipulation techniques currently available for B-splines [22]. Moreover, complex modal patterns can be tackled in a straightforward way without any user-involvement thanks to the local support feature of B-splines, allowing for low-order global fits while still capturing enough detail for data sets with high curvature [23]. While the latter argument is well known and true for most kind of data, it implies non-trivial modifications to the original methodology, the description of which is one of the main purposes of this paper. The rest of the document is organized as follows: Section 2 briefly reviews the fundamentals of B-splines while Section 3 provides a summarized step-by-step account of the novel method introduced in this paper (B-SAFE), also illustrated by results obtained with a representative thin-walled test structure. A validation of the new approach for the case of a 9.2 m composite wind turbine blade, including a comparative of Poly-SAFE versus B-SAFE, is presented in Section 4. Concluding remarks with an outlook on future work are provided in Section 5.

2. Brief fundamentals of B-splines

A canonical B-spline curve is defined by a set of n parametric functions $\boldsymbol{B}(t) : t \rightarrow \Re^n$ which can be viewed as polynomial segments of the same order (degree plus one) $\boldsymbol{\kappa}$ joined at breakpoints (so-called knots) and having $\boldsymbol{C}^e$ continuity conditions [21]. In contrast to power series, the magnitude of the B-spline's fitting error can be reduced by increasing the number of polynomial segments while maintaining the same global order of the overall function [20], allowing datasets with high-curvature regions to be accurately reproduced by low-order (i.e. 3rd) polynomial segments. A B-spline curve defined in a Cartesian coordinate system ς, ϕ can be written as:

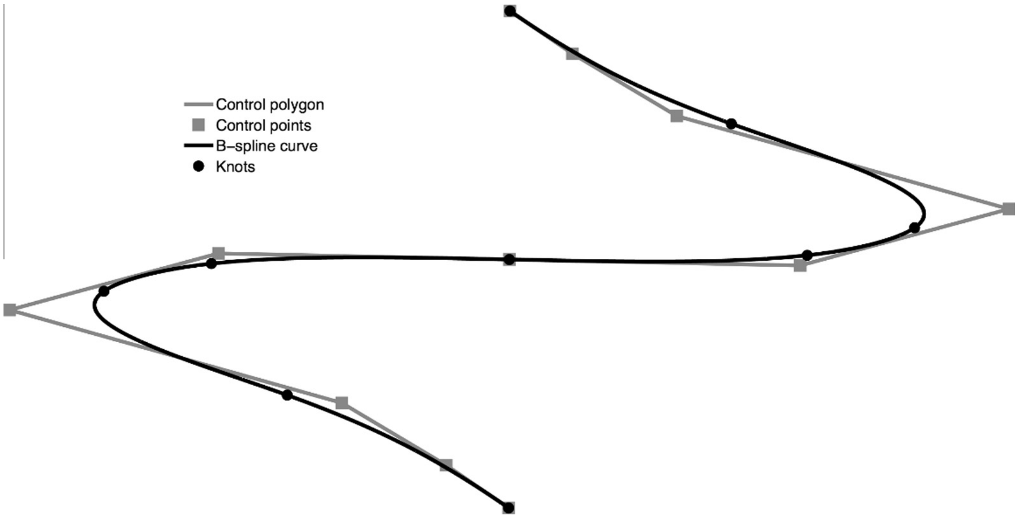

Fig. 1. A cubic B-spline curve with the corresponding control polygon and knots.

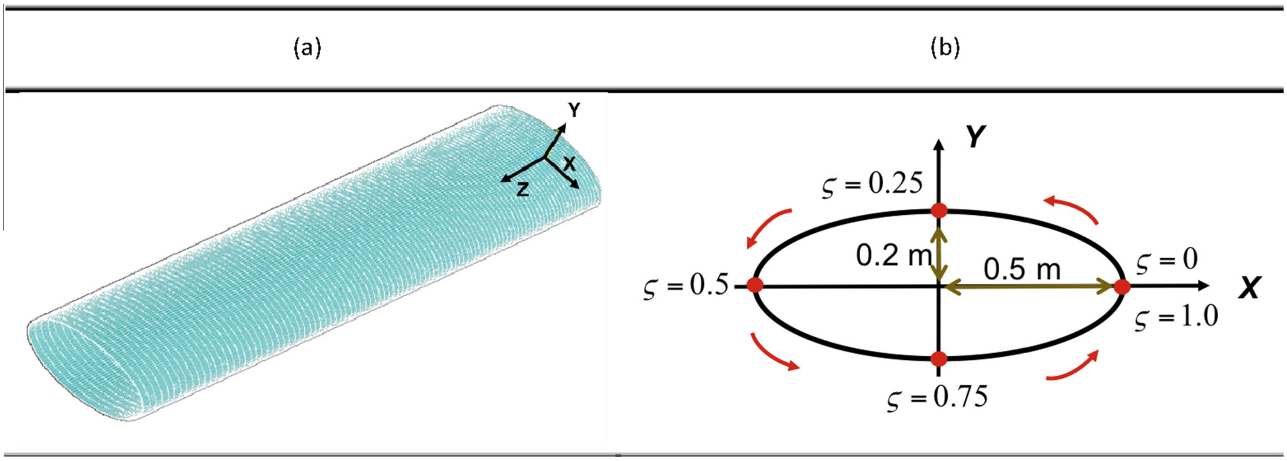

Fig. 2. (a) Finite element model of a thin-walled beam structure with elliptic cross-section, and (b) a generic slice (or cross-section) of the model, which normalized perimeter starts at $\varsigma = 0$ at the right end, proceeding in a counter-clockwise direction and ending at $\varsigma = 1$ at the same initial point.