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Flapwise vibration of rotating composite beams

Tolga Aksencer, Metin Aydogdu *

Department of Mechanical Engineering, Trakya University, 22180 Edirne, Turkey

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ABSTRACT

In the present study, vibration of rotating composite beams is studied. Different beam theories are used in the formulation including Euler–Bernoulli, Timoshenko and Reddy beam theories. Ritz method is used in the solution of the problem. Simple polynomials are chosen for the displacement field. The continuity of transverse stresses is satisfied among the layers. Results are obtained for different orthotropy ratios, rotation speed, hub ratio, length to thickness ratio of the rotating composite beam and different boundary conditions.

1. Introduction

Composite structures are preferred in many engineering applications due to their high stiffness and low density. Different static and dynamic conditions are encountered in their applications. Rotating structures are one important example of composite beams and plates in industrial applications.

Some studies can be obtained related to vibration of rotating structures. Southwell and Gough [1] examined characteristics of the vibration rotating cantilever beams using the energy method. The same problem has been considered by some researchers in the previous studies [2–5]. In 1977 Giurgiutiu and Stafford [6] developed the equations of motion, including shear and rotary of inertia the vibration of the blades at constant angular velocity [6]. Patel et al. [7] have used finite element method in order to investigate the vibration of composite beam. Yoo et al. [8] used Timoshenko beam theory in order to examine behavior of flapwise vibration of rotating multilayer composite beam. Lee et al. [9] investigated the rotating cantilever beam vibration. Ozdemir and Kaya [10] investigated the vibration of rotating tapered cantilever beam. They obtained the natural frequency of a tapered beam by the differential transformation method. In another study, Kim et al. [11] studied vibration of rotating beams. In the same study, axial, chordwise and flapwise motion was derived by using Hamilton principles. Vibration of rotating isotropic and composite uniform and tapered beams was investigated by Ozgumus and Kaya [12]. Vibration of composite beams have been considered in some of the previous studies [13–19].

Although there are some studies related with rotating composite beams higher order shear deformation theories have rarely been considered in the previous studies. The continuity of shear stress also has not been considered in open literature.

The object of this study is to examine the vibration of rotating composite beams using different beam theories including EB, Timoshenko and Reddy theories. The continuity of the shear stresses has been considered among the layers. Ritz method has been used in the solution of the problem. Different material, geometrical and rotational properties are considered.

2. Analysis

A composite beam with dimensions L (length), h (height) and b (width) is taken account (Fig. 1). The beam is constructed of linearly elastic transversely isotropic layers. The composite beam is rotating about an axis with a given angular velocity Ω. A typical rotating composite beam is shown in Fig. 1. The stress in each layer can be given as [20–22]:

$$\sigma_x^{(\chi)} = Q_{11}^{(\chi)}\varepsilon_x, \quad \tau_{xz}^{(\chi)} = Q_{55}^{(\chi)}\gamma_{xz}, \tag{1}$$

where $Q_{ij}^{(\chi)}$ are the reduced stiffnesses [23] and X is the number of layers. By considering shear deformations the following displacement field can be written:

$$\begin{aligned} U(x,z;t) &= u_0(x;t) - zw_{,x} + s(z)u_1(x;t), \\ V(x,z;t) &= 0, \\ W(x,z;t) &= w_0(x;t), \end{aligned} \tag{2}$$

where U, V and W are the displacement components of the beam along the x, y and z directions respectively. u_1 is a function denoting

* Corresponding author. Fax: +90 284 2126067.
E-mail addresses: metina@trakya.edu.tr, metaydogdu@gmail.com (M. Aydogdu).

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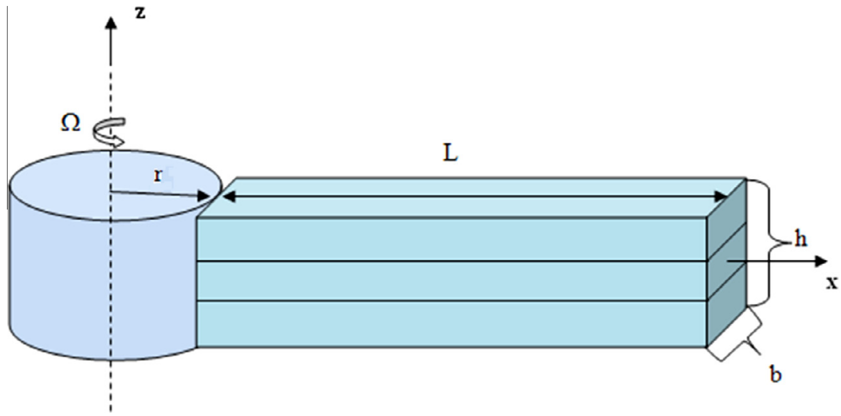

Fig. 1. Geometry and dimensions of rotating composite cantilever beam.

the shear deformation at mid-plane of the beam. Following kinematic relations can be written using Eq. (2):

$$\varepsilon_x = u_{,x} - zw_{,xx} + s(z)u_{1,x}, \quad \gamma_{xz} = s'u_1, \tag{3}$$

where a prime and "x" denote the derivative with respect to z and partial derivative with respect to x respectively.

Parabolic shear deformation beam theory (PSDBT) of Reddy [24], the first order shear deformation beam theory (FSDBT) of Mindlin [25] and Euler–Bernoulli beam theory will be used in this study. In order to do this shape function $s(z)$ should be chosen in the following forms:

$$\begin{aligned} &EBT: s(z) = 0 \\ &FSDBT: s(z) = z, \\ &PSDBT: s(z) = z(1 - 4z^2/3h^2). \end{aligned} \tag{4}$$

The force and moment resultants of the present theory can be defined as:

$$\begin{aligned} &(N_x^c) = \int_{-h/2}^{h/2} (\sigma_x) dz, \quad (M_x^c) = \int_{-h/2}^{h/2} \sigma_x z dz, \\ &(M_x^a) = \int_{-h/2}^{h/2} (\sigma_x) s(z) dz, \quad (Q_x^a) = \int_{-h/2}^{h/2} (\tau_{xz}) s'(z) dz \end{aligned} \tag{5}$$

where superscript 'c' is used for the classical beam theory; whereas superscript 'a' is used for additional quantities incorporating the transverse shear deformation effects. Using Eqs. (1)–(4) the following constitutive equations are obtained [21,22]:

$$\begin{bmatrix} N_x^c \\ M_x^c \\ M_x^a \end{bmatrix} = \begin{bmatrix} A_{11} & B_{11} & E_{11} \\ & D_{11} & F_{11} \\ Sym & H_{11} & \end{bmatrix} \begin{bmatrix} u_{,x} \\ -w_{,xx} \\ u_{1,x} \end{bmatrix} [Q_x^a] = [A_{55}][u_1] \tag{6}$$

The extensional, coupling, bending and transverse shear rigidities are defined as follows:

$$\begin{aligned} &A_{i11} = \int_{-h/2}^{h/2} Q_{11}^{(\chi)} dz, \quad A_{55} = k\int_{-h/2}^{h/2} Q_{55}^{(\chi)} (s')^2 dz, \\ &B_{11} = \int_{-h/2}^{h/2} Q_{ij}^{(\chi)} z dz, \quad E_{11} = \int_{-h/2}^{h/2} Q_{ij}^{(\chi)} s(z) dz, \\ &D_{11} = \int_{-h/2}^{h/2} Q_{ij}^{(\chi)} z^2 dz, \quad F_{11} = \int_{-h/2}^{h/2} Q_{ij}^{(\chi)} s(z) z dz, \\ &H_{11} = \int_{-h/2}^{h/2} Q_{ij}^{(\chi)} (s)^2 dz \\ &(\ )' = d(\ )/dz. \end{aligned} \tag{7}$$

$k = 5/6$ is assumed for the FSDBT.

Application of the Hamilton's principle leads to following equations of motion:

$$\begin{aligned} &N_{x,x}^c = (\rho_0 u + \rho_{01} u_1 - \rho_1 w_{,x})_{,tt}, \\ &M_{x,xx}^c = (\rho_0 w + \rho_1 u_{,x} + \rho_{11} u_{1,x} - \rho_2 w_{,xx})_{,tt} - (F(x) w_{,x})_{,x} + N_x^e w_{,xx}, \\ &M_{x,x}^a - Q_x^a = (\rho_{01} u + \rho_{02} u_1 - \rho_{11} w_{,x})_{,tt}, \end{aligned} \tag{8}$$

where $_{,tt}$ denotes time derivatives and $F(x)$ is the centrifugal force due to rotation which is defined as:

$$F(x) = \frac{1}{2}\int_x^L \rho\Omega^2 (r + x) dx \tag{9}$$

where r is the hub radius shown in Fig. 1 and the ρ's are defined as

$$\begin{aligned} &\rho_i = \int_{-h/2}^{h/2} \rho z^i dz, \quad (i = 0, 1, 2), \\ &\rho_{jm} = \int_{-h/2}^{h/2} \rho z^j f_j^m dz, \quad (j = 0, 1; m = 1, 2) \end{aligned} \tag{10}$$

For the present unified shear deformation composite beam theory, the boundary conditions of the rotating composite beam can be written as:

$$\begin{aligned} &\text{at } x = 0, L \\ &u = \bar{u} \text{ or } N_x^c = \bar{N}_x^c, \\ &w = \bar{w} \text{ or } M_{x,x}^c = \bar{M}_{x,x}^c \\ &w_{,x} = \bar{w}_{,x} \text{ or } M_x^c = \bar{M}_x^c \\ &u_1 = \bar{u}_1 \text{ or } M_x^a = \bar{M}_x^a \end{aligned} \tag{11}$$

where $\bar{u}$, $\bar{w}$, $\bar{w}_{,x}$, $\bar{u}_1$, $\bar{N}_x^c$, $\bar{M}_{x,x}^c$, $\bar{M}_x^c$, $\bar{M}_x^a$ are the prescribed values at the edges of the composite beam.

2.1. The transverse continuity conditions for the symmetric cross-ply beams

By suitable changing the previous shape functions $s(z)$ given in Eq. (2), the continuity of transverse shear strain can be satisfied. Details of this manipulation have been given in the previous works [21,22]. Only the final form of the new shape function that satisfies the transverse continuity conditions is described here.

$$S(z) = A_\chi s(z) + B_\chi \tag{12}$$

where

$$A_\chi = \frac{Q_{55}^{(\chi\mp1)} s'(\chi \mp 1)(z_\chi)}{Q_{55}^{(\chi)} s'(\chi)(z_\chi)} A_{\chi\mp1}, \quad A_0 = 1, \tag{13a}$$