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Spectral element modeling and analysis of the transverse vibration of a laminated composite plate

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ABSTRACT

This paper presents a frequency-domain spectral element model for the symmetric laminated composite plates which have finite dimensions in two orthogonal directions, i.e., in the *x*- and *y*-directions. The spectral element model is developed by using two methods in combination: the splitting of original boundary conditions and the so-called super spectral element method in which both the Kantorovich method-based finite strip element method and the frequency-domain waveguide method are utilized. The present spectral element model has nodes (or degrees of freedom (DOFs)) only on four edges of a finite element, i.e., no nodes inside the finite element. Accordingly the total number of DOFs used in the dynamic analysis can be drastically decreased to lead to a significant decrease of the computation cost, when compared with the standard finite element method (FEM). The high accuracy of the present spectral element model is verified in due course by the comparison with the results by two solution methods: the exact theory available in the literature and the standard finite element model developed in this study.

1. Introduction

It has been well-recognized that composite materials have many advantages over metallic materials due to their high strength-to-density ratios. Thus composite materials have been increasingly applied in the various fields of engineering including aerospace, mechanical engineering, and civil engineering over the past decades [1]. However, the laminated composite structures are susceptible to various modes of damage including delamination. Therefore, it is very important to accurately estimate the dynamic characteristics of the laminated composite structures during the early design phase.

Although the subject of the dynamic analysis of laminated composite plates (simply composite plates) has a quite long history, most existing analytical solutions to this classical problem are considered to be at best approximate because it is difficult to obtain exact closed-form solutions which simultaneously satisfy the governing differential equations and the associated boundary conditions, except for very specific types of plate such as the Levy-type plates [2].

The finite element method (FEM) is certainly one of the most powerful computational methods to analyze the dynamics of a structure with very complex geometry, material distribution, and boundary conditions. The accuracy of FEM depends on the size of finite elements (or meshes) used in the analysis because the displacement fields in a finite element are normally represented by simple polynomial functions which are not dependent of vibrating frequency. Accordingly, as a drawback of FEM, very fine meshing is required to improve the solution accuracy especially at high frequency: this may result in a significant increase of computation cost. Thus, the spectral element method (SEM) can be considered as an alternative to FEM because it can provide extremely accurate solutions even at very high frequencies by using the so-called spectral element matrix (or dynamic stiffness matrix) formulated from frequency-dependent (dynamic) shape functions.

In the literature, there are two completely different methods which have been called the same name 'SEM'. The first one is the frequency-domain SEM [3,4], where accurately formulated dynamic stiffness matrix is used as the finite element stiffness matrix. The second one is the time-domain SEM proposed by Patera [5]. The Patera's time-domain SEM is of course formulated in the time domain by using the Legendre or Chebyshev orthogonal polynomials as the shape functions in conjunction with the use of the Gauss–Lobatto–Legendre integration rule. Thus, the word 'spectral' is time-wise for the frequency-domain SEM, while it is space-wise for the time-domain SEM. The SEM considered in this

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study is the frequency-domain SEM and it will be called 'SEM' throughout the paper.

As mentioned previously, the spectral element matrix (or dynamic stiffness matrix) for SEM is formulated from free wave solutions that satisfy the governing differential equations transformed in the frequency domain [4]. Thus the SEM can provide extremely accurate solutions by representing a whole uniform structure member as a single finite element, regardless of its size, and it has been often recognized as an exact element method. The assembly of finite elements to a global structure system can be conducted in an exactly analogous way as that used in the standard FEM.

Despite of outstanding features of SEM, however its applications have been limited mostly to one-dimensional (1D) structures (e.g., [3,4]) or the plates with very specific geometries and boundary conditions (e.g., [6–18]). In the literature, there are very few spectral element models for two-dimensional (2D) structures which have finite dimensions in both x and y directions and being subjected to arbitrary boundary conditions. This is because, for such finite 2D structures, it is not easy to obtain exact free wave solutions in analytical forms which are essentially required to formulate spectral element matrix. Only a few researchers [19–20] have presented approximate dynamic stiffness methods or spectral element methods for the in-plane or transverse vibrations of the finite plates. Recently, Park et al. [21] presented a spectral element model for a rectangular membrane by using two methods in combination to obtain the displacement field in a finite membrane element: (1) the splitting of boundary conditions; (2) the super spectral element method (SSEM) in which the Kantorovich method-based finite strip element method and the frequency-domain waveguide method are utilized.

As an extension of the author's previous work [21], this paper proposes a spectral element model for a symmetric composite plate. The proposed spectral element model is formulated by using the splitting of boundary conditions and the SSEM in combination. Though the spectral element modeling method newly proposed in this paper can be applied to various laminated plate theories including the classical laminated plate theory (CLPT) and the improved theory such as the first-order shear deformation plate theory (FSDT), the discussion in this paper is limited to the CLPT. The performance of the proposed spectral element model is then evaluated in due course by the comparison with exact solutions and the solutions by the standard finite element model developed in this study.

2. Governing equations

2.1. Governing equations in the time domain

Consider a symmetric composite plate whose layup is symmetric about the midplane of the plate. The composite plate is rectangular and its dimensions in the x- and y-directions are l_x and l_y, respectively. The origin at coordinates (x,y) is placed at the middle of the composite plate, as shown in Fig. 1.

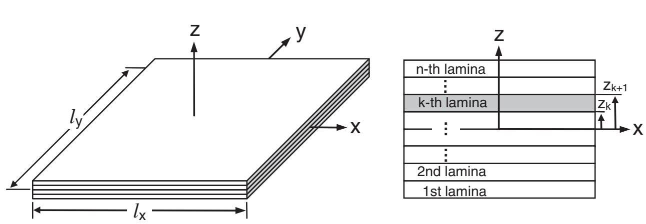

Fig. 1. The geometry of a symmetric laminated composite plate.

The governing differential equation of motion for the transverse vibration of the symmetric composite plate is given by [1]

$$D_{11}\frac{\partial^4 w}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}+4D_{16}\frac{\partial^4 w}{\partial x^3\partial y}+4D_{26}\frac{\partial^4 w}{\partial x\partial y^3}+D_{22}\frac{\partial^4 w}{\partial y^4}+\rho\frac{\partial^2 w}{\partial t^2}=f(x,y,t) \tag{1}$$

where $w(x,y,t)$ is the transverse displacement, $f(x,y,t)$ is the external force applied normal to the surface of the plate, ρ is the mass per unit area of the plate, and D_{ij} are the bending stiffnesses defined by

$$D_{ij}=\frac{1}{3}\sum_{k=1}^{L}\bar{Q}_{ij}^{(k)}(z_{k+1}^3-z_k^3)\quad(i,j=1,2,6,\quad k=1,2,\ldots,L) \tag{2}$$

where $\bar{Q}_{ij}^{(k)}$ are the transformed plane stress-reduced stiffness coefficients of the kth orthotropic layer, (z_k,z_{k+1}) denote the thickness coordinates of the bottom and top, respectively, of the kth layer, and L denotes the total number of layers.

The boundary conditions associated with Eq. (1) are given by

$$\begin{aligned}
&M_y(x,-l_y/2,t)=-M_{y1}(x,t)\quad\text{or}\quad\theta_y(x,-l_y/2,t)=\theta_{y1}(x,t)\\
&V_y(x,-l_y/2,t)=-V_{y1}(x,t)\quad\text{or}\quad w(x,-l_y/2,t)=w_1(x,t)\\
&M_y(x,l_y/2,t)=M_{y2}(x,t)\quad\text{or}\quad\theta_y(x,l_y/2,t)=\theta_{y2}(x,t)\\
&V_y(x,l_y/2,t)=V_{y2}(x,t)\quad\text{or}\quad w(x,l_y/2,t)=w_2(x,t)\\
&M_x(-l_x/2,y,t)=-M_{x1}(y,t)\quad\text{or}\quad\theta_x(-l_x/2,y,t)=\theta_{x1}(y,t)\\
&V_x(-l_x/2,y,t)=-V_{x1}(y,t)\quad\text{or}\quad w(-l_x/2,y,t)=w_3(y,t)\\
&M_x(l_x/2,y,t)=M_{x2}(y,t)\quad\text{or}\quad\theta_x(l_x/2,y,t)=\theta_{x2}(y,t)\\
&V_x(l_x/2,y,t)=V_{x2}(y,t)\quad\text{or}\quad w(l/2_x,y,t)=w_4(y,t)
\end{aligned} \tag{3}$$

where (M_x,M_y) are the resultant bending moments and (V_x,V_y) are the resultant transverse shear forces defined by

$$\begin{aligned}
M_x(x,y,t)&=-D_{11}\frac{\partial^2 w}{\partial x^2}-D_{12}\frac{\partial^2 w}{\partial y^2}-2D_{16}\frac{\partial^2 w}{\partial x\partial y},\\
M_y(x,y,t)&=-D_{12}\frac{\partial^2 w}{\partial x^2}-D_{22}\frac{\partial^2 w}{\partial y^2}-2D_{26}\frac{\partial^2 w}{\partial x\partial y}\\
V_x(x,y,t)&=-\frac{\partial}{\partial x}\left(D_{11}\frac{\partial^2 w}{\partial x^2}+D_{12}\frac{\partial^2 w}{\partial y^2}+2D_{16}\frac{\partial^2 w}{\partial x\partial y}\right)\\
&\quad-\frac{\partial}{\partial y}\left(D_{16}\frac{\partial^2 w}{\partial x^2}+D_{26}\frac{\partial^2 w}{\partial y^2}+2D_{66}\frac{\partial^2 w}{\partial x\partial y}\right)\\
V_y(x,y,t)&=-\frac{\partial}{\partial y}\left(D_{12}\frac{\partial^2 w}{\partial x^2}+D_{22}\frac{\partial^2 w}{\partial y^2}+2D_{26}\frac{\partial^2 w}{\partial x\partial y}\right)\\
&\quad-\frac{\partial}{\partial x}\left(D_{16}\frac{\partial^2 w}{\partial x^2}+D_{26}\frac{\partial^2 w}{\partial y^2}+2D_{66}\frac{\partial^2 w}{\partial x\partial y}\right)
\end{aligned} \tag{4}$$

and θ_x and θ_y are the slopes defined by

$$\theta_x=\frac{\partial w}{\partial x},\quad\theta_y=\frac{\partial w}{\partial y} \tag{5}$$

2.2. Governing equations in the frequency domain

To formulate spectral element model for a composite plate by following the general procedure introduced in Ref. [4], firstly all the time domain quantities in the governing differential equations of motion (Eq. (1)) and the boundary conditions (Eq. (3)) are transformed into the frequency domain quantities by using the discrete Fourier transform (DFT) theory [22]. For instance, the transverse displacement $w(x,y,t)$ and the external force $f(x,y,t)$ can be represented in the spectral forms as