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Analytical sensitivity analysis of eigenvalues and lightweight design of composite laminated beams

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A R T I C L E I N F O

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A B S T R A C T

The paper describes an optimisation technique for the lightweight design of composite laminated beams with a big ratio of span to thickness. The optimisation model for the lightweight design is to find the fibre volume fractions to minimise the mass of the composite laminated beams under the eigenvalue (or frequency) constraints. The analytical sensitivity of the eigenvalues, with respect to the fibre volume fractions, is formulated using the Euler–Bernoulli beam theory. The analytical formulae for sensitivity analysis are suitable for the different boundary conditions (pinned–pinned, fixed–fixed, fixed–free and fixed–pinned). The eigenvalues are linearised using Taylor's series so that optimisation model is converted to a linear programming problem. The iterative computational procedure is proposed to solve the optimisation model and eliminate the Taylor's series approximation since the current design point maybe not near the optimum design. Finally, the lightweight designs of the composite laminated beams with different boundary conditions are performed. The merits of the proposed method and lightweight designs obtained by proposed method are discussed.

1. Introduction

Composite laminated structures are gaining more and more applications in aircraft, automobile, naval and defence industries because of their high performance characteristics, such as high strength-to-weight ratio, high stiffness-to-weight ratio, superior fatigue properties and high corrosion resistance [1,2]. As the research, knowledge and confidence on the composite structures increases, the composite structures are gradually acting as the main load-carrying components, not only as the secondary load-carrying components in the engineering application. The beams are the major lateral load-carrying member in an engineering structure system. Compared with the beam structures made of single isotropic material, such as metal beams, composite beam structures produce bigger challenge in structural analysis because the mechanical properties of the composite material is anisotropic, and however allow the engineers to design the materials, not just the size and shape in structural design. The composite laminated beams are very promising in the vibration environment because the lightweight material attracts smaller inertia force than heavy material, such as structural steel. Many researchers have investigated the vibration characteristics of the composite laminated beams. From the viewpoint of the number of independent displacements, the composite beam has the following vibration models (the coordinated system is shown in Fig. 1): ① middle plane displacement w (z direction) [3]; ② middle plane displacement w (z direction) and middle plane bending slope ψ_x [4]; ③ middle plane displacement w (z direction), middle plane displacement u (x direction), middle plane bending slopes ψ_x and ψ_y [5]; ④ middle plane displacement w (z direction) and torsional rotation angle ϕ_x around x axis [6,7]; ⑤ middle plane displacement w (z direction), torsional rotation angle ϕ_x around x axis and middle plane bending slope ψ_x [8]. The models ①, ② and ③ may be called Euler–Bernoulli beam model, Timoshenko beam model and general beam model, respectively. The models ④ and ⑤ may be called bending–torsion beam model and bending–torsion coupled Timoshenko beam model. The composite beam models ①, ② and ③ are direct based on the composite laminate theory. However, the bending rigidity, torsional rigidity and bending–torsion material coupling rigidity of composite beams are necessary in the composite beam models ④ and ⑤. The number of the independent displacements leads to the same number of the partial differential equations. The results in literature [4] indicate that for a composite laminated beam with a big ratio of span to thickness, the different beam models can lead to the same eigenvalues (natural frequencies). Like the metal beams, the shear deformation and rotary inertia can be

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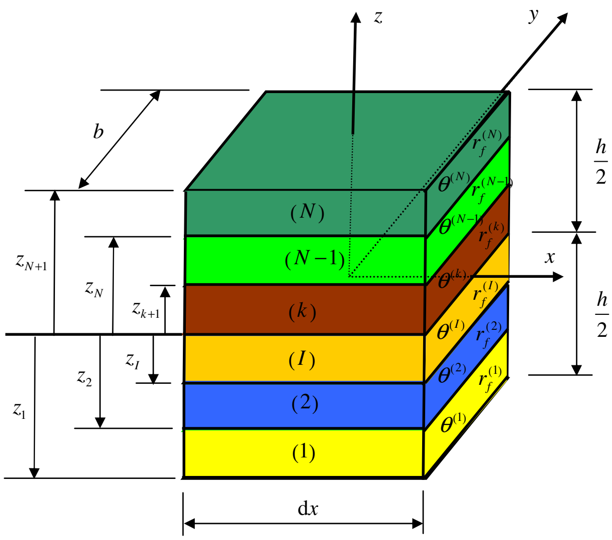

Fig. 1. Composite laminated beam (segment dx).

neglected in the composite laminated beams with a big ratio of span to thickness.

The discrete models (finite element models) are often employed to analyse and optimise the composite beam structures because the finite element models are believed to be easier implemented than the continuous composite beam models. Many researchers developed the analysis and optimal design methods for the composite beam structures using discrete models. For example, Cardoso et al. [9,10] used finite element technique to deal with design sensitivity and optimal design of composite thin-walled laminated beams using torsion–bending beam model. Neto et al. [11] performed the sensitivity analysis and optimal design of composite beam structures using finite element solver FEAP. Sedaghati et al. [12] developed a finite element model to study the mechanical and electrical behaviour of laminated composite beam with piezoelectric actuators and a design optimisation methodology has been developed by combining the finite element model and the sequential quadratic programming technique. Blasques and Stolpe [13] performed the maximum stiffness and minimum weight optimisation of laminated composite beams using finite element approach. The fibre orientations and layer thicknesses are design variables. Hamdaoui et al. [14] investigated an optimal design approach for choosing the most suitable material for high damping and low mass for a sandwich beam. Valido and Cardoso [15] implemented the optimal design of the various geometrically nonlinear composite laminate beam structures based on finite element analysis and sensitivity analysis model. However, the recent research indicates that the numerical values of the eigenvalues and their sensitivity obtained by the finite element models heavily depend on the mesh schedules [16]. On the other hand, the finite element approaches have much lower efficiency than the analytical methods. Many researchers have developed the analytical methods for the analysis of composite structures. For example, the analytical methods have been developed to analyse the free vibration problem of the composite beams [3–8]. The analytical analysis methods and optimisation design using different non gradient-based algorithms for the thin-walled composite box-beam helicopter rotor blame have been investigated [17–20]. From the viewpoint of the design variables, the design variables chosen in the previous works are fibre orientations (stacking sequences), thickness of the layers, cross-sectional dimensions. However, the fibre volume fractions have not been chosen as the design variables. The work [16] indicates that the stiffness, mass, eigenvalues and eigenvectors of the composite laminated structures heavily depend on the fibre volume fractions.

The purpose of this paper is to develop a lightweight design method for the composite laminated beams with a big ratio of span to thickness. The analytical sensitivity of the eigenvalues, with respect to the fibre volume fractions, is derived based on the Euler–Bernoulli beam theory. For the given fibre orientations and thickness of layers, the lightweight design optimisation model is to minimise the mass of the composite laminated beams so that the eigenvalue (or frequency) and fibre volume fraction constraints are satisfied. The eigenvalues are linearised using Taylor's series and the lightweight design optimisation model is converted to a linear programming problem. The iterative algorithm is proposed to eliminate the Taylor's series approximation since the current design point maybe not near the optimum design. Finally, the lightweight designs of the composite laminated beams with different boundary conditions and two given fibre orientations are demonstrated. The merits of the proposed method are also discussed.

2. Free vibration equation of composite laminated beams

A segment dx of composite laminated beam with rectangular cross section is shown in Fig. 1. The laminated beam is made of N layers. The number of layers are denoted by $(1),(2),\ldots,(N)$ and the fibre orientations of layers are denoted by $\theta^{(1)},\theta^{(2)},\ldots,\theta^{(N)}$. The fibre volume fractions of layers are denoted by $r_f^{(1)}, r_f^{(2)},\ldots, r_f^{(N)}$ or $r_1, r_2, \ldots, r_N$ in this paper. The layered positions are denoted by $z_1, z_2, \ldots, z_N, z_{N+1}$. The width and thickness of the rectangular cross section are denoted by b and h, respectively. The fibre volume fractions, fibre orientations and thickness of layers have influences on the characteristics of free vibration of the composite laminated beam. However, in this paper, the fibre orientations and thickness of layers are supposed to be given, only the fibre volume fractions are the design variables of the composite beam so that the reasonable distribution of the fibre volume fractions at different layers can be found.

The free vibration equation of the composite laminated beams based on Euler–Bernoulli beam theory can be expressed as

$$bD_{11}\frac{\partial^4 w}{\partial x^4}+\sum_{k=1}^{N}\rho^{(k)}b(z_{k+1}-z_k)\frac{\partial^2 w}{\partial t^2}=0 \tag{1}$$

The width b can be divided out of the above free vibration equation.

$$D_{11}\frac{\partial^4 w}{\partial x^4}+\sum_{k=1}^{N}\rho^{(k)}(z_{k+1}-z_k)\frac{\partial^2 w}{\partial t^2}=0 \tag{2}$$

where w is the deflection of the composite laminated beam. D_{11} is called bending stiffness. $\rho^{(k)}$ is the mass density of the kth layer. t is the time coordinate. The explicit expression of $\rho^{(k)}$ is

$$\rho^{(k)}=r_f^{(k)}\rho_f+\left(1-r_f^{(k)}\right)\rho_m \tag{3}$$

where ρ_f and ρ_m are the mass density of the fibre and matrix, respectively. The bending stiffness D_{11} can be computed based on the classic composite laminate theory, i.e.,

$$D_{11}=\frac{1}{3}\sum_{k=1}^{N}\bar{Q}_{11}^{(k)}\left(z_{k+1}^3-z_k^3\right) \tag{4}$$

where $\bar{Q}_{11}^{(k)}$ is called plane stiffness coefficients of the kth lamina in the laminate coordinate system.

$$\bar{Q}_{11}^{(k)}=Q_{11}^{(k)}\cos^4\theta^{(k)}+2\left(Q_{12}^{(k)}+2Q_{66}^{(k)}\right)\sin^2\theta^{(k)}\cos^2\theta^{(k)}+Q_{22}^{(k)}\sin^4\theta^{(k)} \tag{5}$$