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Modeling of magnetoelectric composite nano-cantilever beam with surface effect

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ABSTRACT

This work investigates the magnetoelectric response of a magnetostrictive–piezoelectric composite nano-cantilever beam with consideration of surface effects through the surface-layer-based model. The magnetoelectric composite nano-cantilever is treated as a bulk core plus two surface layers. The influences on the cantilever's overall properties resulted from the surface effect is modeled as a spring force exerting on the boundary of the bulk core. A Kirchhoff's theory is used to get the explicit solutions for the magnetoelectric effect of cantilever when subjected to bias magnetic field loads. In order to apply the appropriate boundary conditions on the cantilever, the effective axial force, shear force and moment are derived. Using the derived results, the so-called effective Miller–Shenoy coefficient, static and electromechanical resonance properties of the ME composite nano-cantilever beam for the extensional–bending coupling deformations are analyzed theoretically. At the same time, the effect of the substrate on ME effect is theoretically studied by altering the thickness of the substrate. The results indicate that the surface effects play a significant role in the magnetoelectric response of the ME composite nano-cantilever beam. This work is very helpful for design of nano-cantilever beam based devices and understanding the size-dependent properties of nanostructured ME composites in nano-electromechanical systems.

1. Introduction

Multiferroic magnetoelectric (ME) systems represent a new class of materials that have attracted much attention because of their potential for enhanced functionality in sensors and other devices [1]. Their distinctive characteristics are the cross-coupling effect between electric polarization and magnetization which called ME coupling effect. The ME effect is investigated by subjecting the sample to a bias magnetic (electric) field and then measured the resulting electric (magnetic) field produced in the material. Such effect can be achieved through single-phase or composites material systems. However, the ME effect in single-phase materials is usually very small and needs to be observed under very rigid conditions such as at extremely low temperature [2,3]. Therefore, the ME composites have attracted much attention due to their large ME effect. In such composite materials, the ME effect is strain or stress mediated, that is, the strain induced in one component, either by magnetostriction in the magnetostrictive (MS) phase or by the piezoelectric effect in the piezoelectric (PE) phase, is transferred to the other component, altering the polarization or magnetization. Generally, ME composites are including particulate composites, fiber-array composites, and laminate (layer) composites [1]. In particular, laminate ME composites have opened up opportunities to obtain enhanced ME coefficient due to the higher coupling effect and reduction in the charge leakage of the piezoelectric phase. With the rapid development of modern nanoscience and nanotechnology, magnetoelectric nanostructures have attracted tremendous attention due to their potential applications in novel ME nano-devices [4–9]. These novel structures, i.e. thin film, enables the design of more complicated and fascinating devices in nano ME systems. To further explore the design and application possibilities of ME composite nanostructured materials, it is essential to fully understand their magneto-elasto-electric coupling behaviors and predict their ME responses.

It is well known that the miniaturization of material into a nanoscale size will significantly increase the surface-to-volume ratio. It thus may substantially affect the physical and mechanical properties of the nano-films. This presumption has already

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confirmed for elastic nano-materials where significant surface effect leads to size-dependent elastic properties and the vibration and bulking behaviors distinct from those of their bulk counterparts. Detailed characterization of the surface behavior of solids often required extensive atomistic analysis and/or experimental tests. Therefore, much theoretical and experimental effort has been directed toward the investigation of surface effects on mechanical behavior of nano-structures. In addition to atomic studies, researchers have accounted for surface effects in continuum modeling using the linear surface elastic theory developed by Gurtin and Murdoch, is the called GM theory [10,11]. Under the GM theory assumption, a surface can be regarded as a thin layer with negligible thickness perfected bonded to the bulk without slipping; and such analysis model is known as the surface layer model. Such a surface-layer-based model has been widely adopted to investigate the nanostructured materials. As an extension of the surface elasticity model, Huang and Yu developed a surface piezoelectricity model, which incorporates surface piezoelectricity in addition to the surface elasticity and residual surface stress, to study the mechanical and electrical response of a piezoelectric nano-structure [12]. It was found that the surface effect influences the electromechanical coupling behaviors of piezoelectric nano-structures size-dependent and significantly.

ME coupling effect is arising from elastic mechanical coupling between the constitutive phases, i.e. magnetostrictive and piezoelectric. In order to choose suitable components and structure designing to optimal the ME response, a number of models have been developed to study the ME effect of the nano ME system based on conventional continuum theories [7,13–17]. For the ME composites, the surface effect may be also given a significant effect on its overall properties at the nano-scale. Therefore, capturing the nature of such surface effect is a new challenge in theoretical modeling of ME nanostructures. Hao et al. firstly studies the static behaviors of ME nano plates including the effects of flexural strains and residual surface stress [18]. Recently, Shi et al. studies the influence of the surface effects on the electromechanical resonant coupling and static bending of magnetostrictive-piezoelectric nano-bilayer on a substrate by considering the combined the effects of surface elastic, surface piezoelectricity, surface piezomagneticity, and residual surface stress [19]. Their model adopted nonlinearly constitutive equations for MS layer and neglected the effect of surface properties on the neutral plane, the positions of which depends on the thickness and module of elastic beam as well as the surface module. Although many researchers on nano-structures have conducted, most of them were not successful in deriving the ME coupling effect with surface effect by using a more accurate theoretical model.

The objective of this paper is to establish modeling equations of ME composites with nano-thickness, in which the surface effect is taken into account. The surface effects are taken into account by using the nonclassical boundary conditions in which some sort of spring force is acting on the bulk core. The basic equations of nano-scale ME composite are derived, which automatically include the surface effect. Using the derived equations, the static and electromechanical resonant (EMR) properties of ME effect for nano-cantilever beam are analyzed. Furthermore, the effect of the substrate is also taken into consideration.

2. Basic equations

Consider a ME composite nano-cantilever beam with magnetostrictive and piezoelectric layers on opposite sides of the substrate as shown in Fig. 1. The thickness of layer 1 (magnetostrictive layer), layer 2 (substrate layer), and layer 3 (piezoelectric layer) are denoted as t_m, t_s, and t_p, respectively. A Cartesian coordinate system x_i ($i = 1,2,3$) is introduced so that the axes x_1 and x_2 are coordinate lying in the length–width plane, x_3 is orthogonal to them. The x_3-coordinate of upper and lower surface of kth layer is denoted by z_k and z_{k-1}, respectively. The layers are perfectly bonded together, and the interface effect is not considered in this paper. The surface layer and the bulk core have different material properties. Due to the asymmetric characteristic of the cantilever composites, non-uniform distributed strain causes extensional deformation as well as flexural deformation. The extensional deformation and flexural deformation are coupled and influenced each other. In this case, there is no middle plane of the beam that can serve as a plane of symmetry. It is well known that in the asymmetrical structures, a neutral plane, the position of which depends on the geometry size and material parameters as well as the field distribution, plays a similar role.

For the bulk core of the piezoelectric layer, the constitutive equations are the same as the classical piezoelectricity [13], i.e.,

$$ {}^pS_{kl} = {}^ps_{ijkl}{}^pT_{ij} + d_{ijk}E_k \tag{1} $$

$$ D_i = d_{ikl}{}^pT_{ij} + \varepsilon_{ik}E_k \tag{2} $$

where T_{ij}, S_{ij}, D_iE_k are stresses, strain, electric displacement and electric fields, and s_{ijkl}, d_{mkl}, and ε_{mn} are compliance, piezoelectric and dielectric constants of the bulk, respectively. The sub index p corresponding to the piezoelectric layer.

For the giant magnetostrictive materials, it has been proved by the experiment that the material parameters such Young's modulus and piezomagnetic coefficient will change with internal stress and external magnetic field, which is called the nonlinear magnetostrictive effect. For simplicity, we use the equivalent linear constitutive relations instead of the nonlinear magnetostrictive relations. Here, the material constants in this linear constitutive equations will change with bias magnetic field and pre-stress. The equivalent linear constitutive equations for the bulk core of magnetostrictive layer are as follows [20]

$$ {}^mS_{kl} = {}^ms_{ijkl}({}^mT_{ij}, H_k){}^mT_{ij} + q_{ijk}({}^mT_{ij}, H_k)H_k \tag{3} $$

$$ B_i = q_{ikl}({}^mT_{ij}, H_k){}^mT_{ij} + \mu_{ik}({}^mT_{ij}, H_k)H_k \tag{4} $$

Here, B_k ,H_k,S_{kl}, and T_{ij} are respectively magnetic induction, magnetic field, strain and stress, $s_{ijkl}({}^mT_{ij}, H_k)$, $q_{ijk}({}^mT_{ij}, H_k)$, and $\mu_{ik}({}^mT_{ij}, H_k)$ are respectively the effective elastic compliance, piezomagnetic coefficient and magnetic permeability constants, respectively. The sub index m corresponding to the magnetostrictive layer. Please note that the constants in this linear constitutive relationship will change with bias magnetic field and pre-stress, and the detail expressions of the material constants will discussed in the following.

For the substrate layer, we have to introduce the third constitutive equation as following [8]

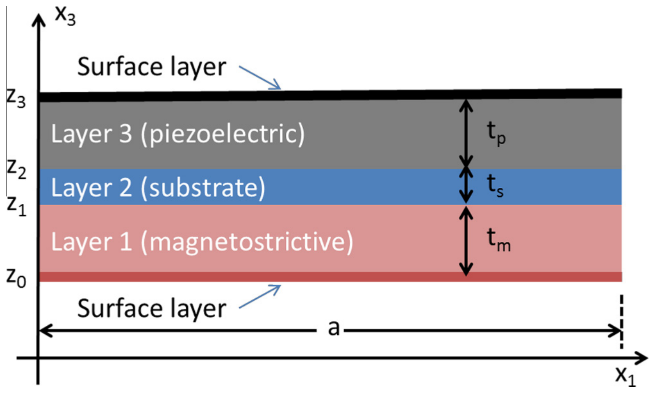

Fig. 1. Schematic sketch of the ME layered composite with surface effect.