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Buckling analysis of functionally graded materials structures with enhanced solid-shell elements and transverse shear correction

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ABSTRACT

In this paper, the buckling behavior of functionally graded material (FGM) structures is investigated using the enhanced assumed strain (EAS) solid-shell element based on first-order shear deformation concept. Thus, a computational algorithm, for any type of laminates composites and/or FGM, is proposed for the shear correction factors estimation. Material properties are varied continuously in the thickness direction according to different distributions. This finite element is used to study the buckling behavior of FGM structures and to investigate the influence of same parameters on the buckling load. Comparisons of numerical results among existing ones show the performance of the developed elements.

1. Introduction

Composite materials are widely used in several engineering fields because of their enhanced mechanical properties. However, they present interface problems due to changes in material properties across the section leading to delamination. To overcome these problems a group of Japanese scientists [1,2] introduced the FGMs "functionally graded materials". The advantage of these materials is the continuous change in their mechanical and thermal properties. The mechanical properties change gradually in the thickness direction according to volume fraction power-law distribution [2,3]. The stress distributions are smooth hence interface problems are eliminated. Few studies have been done in the analysis of buckling behavior of functionally graded materials (FGM) structures. The researchers are divided into two primary classes: the analytical and finite element approaches.

Several analytical studies about the buckling analysis of FGM structures exist in the literature. For instance, Javaheri and Eslami [4,5] carried out mechanical and thermal buckling analyses of FGM plates using Kirchhoff's thin plate theory. The equilibrium and stability equations for a simply supported rectangular plate under different loading conditions are obtained. Based on the Mindlin's plate theory, Lanhe [6] obtained the critical buckling load for a simply supported rectangular plate subjected to two types of thermal loading of moderately thick functionally graded plates. Shariat and Eslami [7] studied mechanical and thermal buckling of simply supported thick rectangular functionally graded plates using the classical and higher-order shear deformation plate theories. The structure is under different loading conditions. Using the element-free kp-Ritz method, Lee et al. [8] studied the post buckling of functionally graded plates under edge compression and temperature field. Thai and Choi [9] presented a refined theory for buckling analysis of functionally graded plates under uniaxial and biaxial compressions. Sofiyev and Schnak [10] studied the stability problem of functionally graded cylindrical shells subjected to torsion varying as a linear function of time, by using the Lagrange–Hamilton type principle. Shen [11] studied the thermal buckling and post buckling of thin and shear deformable functionally graded ceramic–metal cylindrical shells subjected to uniform temperature rise or heat conduction.

Concerning the second approaches to solve FGM structures problems, the finite element method, Natarajan et al. [12] studied the buckling behavior of FGM plates with local defects by using an enriched shear flexible 4-noded element. Using the third-order shear deformation theory (TSDT), Reddy [13] developed a finite element model for the analysis of through-thickness functionally graded plates. Na and Kim [14] investigated the three dimensional thermal buckling analysis of FGM plates using 18 nodes solid element. Asemi et al. [15] used a full compatible three-dimensional Hermitian element with 168 degrees of freedom which guarantees continuity of the strain and stress components to investigate buckling of heterogeneous functionally graded plates under different loading conditions. Using an eight nodes C^0 continuous shear

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flexible plate bending element with five degrees of freedom per node, Prakash et al. [16] studied the thermal post buckling analysis of FGM skew plates. Recently, an efficiency three dimensional double directors shell element for the FGM shell structures analysis is proposed by Wali et al. [17]. The vanishing of transverse shear strains on top and bottom faces is considered in a discrete form inspired form the work of Dammak et al. [18]. Thus, the third-order shear deformation plate theory (TSDT) is a particular case of the discrete double director shell model.

Many researchers have investigated the buckling behavior of FGM structures using diverse approaches and methods. To the authors knowledge, up to now, there are no works has been done on the investigation of FGM structures behavior using enhanced solid-shell elements. One exception is the work of Zheng et al. [19], who study the static and dynamic control of FGM shells subjected to temperature gradient using the enhanced assumed strain piezoelectric solid-shell finite elements based on the assumed natural strain (ANS) and enhanced assumed strain method (EAS).

The solid-shell finite element has been widely used in the literature. Klinkel et al. [20] and Vu-Quoc and Tan [21], among many others, developed three-dimensional locking-free solid-shell elements by using the EAS and ANS methods. In Hajlaoui et al. [22], the authors presented a solid-shell finite element with an ANS method to resolve the shear locking effect and a nine parameters EAS method which avoided completely the volumetric locking to study the buckling behavior of laminated composite plate with delamination.

Since most of the developed solid-shell elements are based on a transverse shear strain calculated at the mid-surface, implying a constant variation through the thickness, this requires the introduction of shear correction factors. Also, to the author's knowledge, there are no shear correction factors introduced in any solid-shell elements developments.

It is well known that in the first-order shear deformation theory (FSDT), the shear distribution across the shell thickness is constant and require the introduction of shear correction factors even for solid-shell element. One can find, in the literature, many proposals formulas for shear correction factors depending on: static, dynamic, materials, and laminates. Most of those formulas are based on the static equilibrium and energy equivalence between the shear energy of the plate or shell and the energy from three-dimensional elasticity theory, [23–26,32,33]. Batoz and Dhatt [23] calculated the shear correction factors for a symmetric cross-ply composite laminated plates which is not suitable for the non-symmetrical FGM plates and shells. Furthermore, procedures for the computation of the through-the-thickness shear stresses together with an iterative algorithm based on shear strain energies equivalence for the evaluation of the shear correction factors are proposed in Auricchio and Sacco [25,26] with a FSDT and in Huang [27] with a TSDT. Predictor–corrector approaches are also used to calculate the shear correction factors in Sze et al. [28]. Another approach was proposed by Pai [29], which is based on a layer-wise higher-order shear deformation theory to propose a formulation of shear correction factors that can be used to obtain accurate estimates of shear correction factors for the use of the first-order shear deformation theory. Altenbach [30], propose the determination of the transverse shear stiffness by comparing the forces and moments solution of static problems from two-dimensional and three dimensional theories. Tanov and Tabiei [31], presented a simple correction to the first-order shear deformation shell finite formulation where they enforced a parabolic shear strain distribution across the shell thickness. In Nguyen et al. [32], the authors propose a simple formula for only one shear correction coefficient of sandwich plate made of isotropic and/or functionally graded materials only. Recently, Shariyat and Alipour [33], propose an analytical expression of the shear correction coefficient in a more general case of an annular functionally graded plate subjected to both normal and shear non-uniform tractions and/or resting on a non-uniform Winkler–Pasternak elastic foundation.

In most of FGM structures, the distribution of constructing materials is not symmetric about the middle surface. In this case the bifurcation buckling did not exist due to the stretching/bending coupling effect, and the bifurcation solutions for FGM rectangular plates with simply supported boundary conditions and/or involving free edges subjected to in-plane uniaxial or biaxial compression or uniform temperature rise are physically incorrect. In order to resolve this problem, Zhang and Zhou [34] used the physical neutral surface theory. Aydogdu [35] used a bending moment for simply supported functionally graded plates to remain flat under in-plane loading. Naderi and Saidi [36] investigated the configuration of the functionally graded plates under in-plane loads lower than the critical buckling load (in the pre-buckling state).

The aim of this paper is to extend the work of Hajlaoui et al. [22] to develop an enhanced assumed solid-shell element based on the assumed natural strain and the enhanced strain method to investigate the behavior of FGM structures under buckling loads. Since FGM material are non-symmetric and it will be experiencing bending at the initial stage of loading under compressive load with simply supported boundary condition, therefore a suitable bending moment is applied to keep the plate flat under in-plane loading before buckling as proposed in Naderi and Saidi [36]. A computational algorithm, based on the static equilibrium and energy equivalence between the shear energy of the shell and the energy from three-dimensional theory, is proposed for the shear correction factors estimation. This shear correction factors computational algorithm can be used not only for FGM shell-like structures, but also for symmetric and non-symmetric orthotropic laminates composites. In this algorithm we have extended the work of Batoz and Dhatt [23] by taking into account the coupling of membrane-bending which is indispensable for FGM structures.

The remainder of this paper is organized as follows. Functionally graded materials are described in Section 2. After that, solid-shell finite element formulation and a general matrix formulation of shear correction factors calculation for any type of laminate, including FGM, is described in Section 3. Numerical results and discussions of the finite element model are investigated in detail in Section 4. Finally, some concluding remarks are analyzed and presented in Section 5.

2. Functionally graded materials

In this section we consider an FGM shell structure made from mixture of the two constituents (metal and ceramic), in which the composition is varied continuously in the thickness direction. Young's modulus $E(z)$ is assumed to vary through the shell thicknesses according to the power-law function (P-FGM), the sigmoid function (S-FGM), or with the exponential function (E-FGM). Poisson's ratio ν is assumed to be constant [7,13,37,38].

2.1. Power functionally graded material (P-FGM)

The material properties for P-FGM of shell-like structures vary continuously through the thickness based on a power-law function as

$$E(z) = E_c g(z) + E_m[1 - g(z)] \quad \text{with} \quad g(z) = \left(\frac{z}{h} + \frac{1}{2}\right)^n \tag{1}$$

where h is the thickness, z is the dimensional coordinate measured normal the shell from the mid-surface and $g(z)$ is the volume fraction of the constituents which can mostly be defined by power-law