Composite Structures 132 (2015) 239–254

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

# Three-dimensional wave propagation on orthotropic cylindrical shells with arbitrary thickness considering state space method

R. Talebitooti [a,*], R. Ahmadi [b], M.H. Shojaeefard [a]

[a] Sch. of Mech. Eng., Iran Univ. of Sci. and Tech., Tehran, Iran
[b] Dept. of Mech. Eng., Payame Noor University (PNU), P.O. Box 19395-3697, Tehran, Iran

ARTICLE INFO

*Article history:*
Available online 15 May 2015

*Keywords:*
Orthotropic shells
Transmission loss
State-space method
Acoustic
Transfer matrix

ABSTRACT

This work presents an analytical solution based on the three-dimensional exact equations of anisotropic elasticity for sound transmission through orthotropic cylindrical shells with arbitrary thickness using a laminate approximate model and a state space method along with the transfer matrix approach. The shell is assumed to be infinitely long and is subjected to an oblique plane wave with uniform external airflow in the external fluid medium. The developed equations are applied to the calculation of the diffuse field transmission of a cylinder. Predictions with the presented models are compared with those of previous models for thin shells. Similar results are observed due to restricted effects of shear and rotation on transmission loss (TL). However, the presented model demonstrates more accurate results for thick shells as the shear and rotation effects become more significant in lower $R/h$ ratio. Additionally, the effects of related parameters on transmission loss such as material and geometrical properties are discussed.

## 1. Introduction

Cylindrical structures are a special class of modern structures, frequently used in the aeronautics, aerospace and automotive industries. Recently, because of strong potential applications of this type of structure, there have been increasing research and development activities in the field of vibration analysis, particularly, noise transmission through such structures. In this study, the case of acoustic transmission into an orthotropic cylinder of infinite length, excited by a diffuse field is considered. This class of problem has been presented, to a certain extent in several previous publications. The acoustic transmission through the circular shell has been studied by Smith [1], White [2], Koval [3–5], Blaise et al. [6–9], Kim et al. [10,11], Daneshjou et al. [12–15] and Cao et al. [16] for isotropic, orthotropic, and laminated fiber-reinforced composite shells.

An analytical study of transmission of sound energy through a thin cylindrically isotropic shell excited by an oblique plane wave was presented by Smith [1]. In his study, a semi-quantitative discussion of noise transmission through a curved structure was presented. White [2] analyzed sound transmission into finite cylindrical shells and determined two important characteristics, the ring frequency and coincidence frequency, which at these two frequencies, the TL obtained the maximum values.

Koval [3,4] presented mathematical models for calculation of the TL of oblique plane sound waves into an orthotropic infinite shell using the displacement field derived by Nelson et al. [17]. In his study, the effects of membrane and bending were considered, but transverse shearing and rotational inertia were neglected. The effect of orthotropic behavior on the sound transmission was parametrically studied for the shell's elastic properties along circumferential and axial directions. In his work, the important key point was taking into account the external airflow, plane wave with an incident angle, and also the internal pressure of the cylindrical shell. The suggested solution was used for the study of the transmission of airborne noise through the isotropic and orthotropic fuselage under flight conditions using the impedance method. Koval [5] later proposed an analytical model for predicting TL of laminated composite infinite cylindrical shells excited by an oblique plane wave.

Blaise et al. [6] then extended Koval's work and considered an orthotropic shell excited by an oblique plane sound wave with two independent incident angles in order to calculate the diffuse field transmission coefficient. They compared the numerical results with Koval's [4] and reported especially some numerical errors in his work. In their study, a Donnell–Mushtari's displacements field is used for orthotropic cylinders. However, the transverse shearing and rotational inertia were completely ignored. They also extended

---

[*] Corresponding author.
*E-mail addresses:* rtalebi@iust.ac.ir (R. Talebitooti), r.ahmadi@albpnu.ac.ir (R. Ahmadi), mhshf@iust.ac.ir (M.H. Shojaeefard).

http://dx.doi.org/10.1016/j.compstruct.2015.05.023
0263-8223/

## Nomenclature

| | |
|---|---|
| $c_3,\ c_1$ | speed of sound in external and cavity medium |
| $E$ | module of elasticity |
| $f_R$ | ring-frequency |
| $f_c$ | critical frequency |
| $G$ | module of rigidity |
| $H_n^1$ | cylindrical Henkel functions of the first kind of integer order $n$ |
| $H_n^2$ | cylindrical Henkel functions of the second kind of integer order $n$ |
| $h$ | shell wall thickness |
| $J_n$ | cylindrical Bessel function of the first kind of order $n$ |
| $k$ | wave number |
| $k_r,\ k_z$ | wave numbers in $r$ and $z$ direction |
| $M_1$ | Mach number |
| $n$ | circumferential mode number |
| $P_0$ | amplitude of the incident wave |
| $p_1^I$ | acoustic pressures of the incident wave |
| $p_1^R$ | acoustic pressures of the reflected wave |
| $p_3^T$ | acoustic pressure of the transmitted wave |
| $Q_{ij}$ | orthotropic stiffness coefficients |
| $(r,\varphi,z)$ | components of cylindrical coordinate |
| $R_0,\ R_p$ | inner and outer radius |
| TL | transmission loss |
| $T_n$ | global modal transfer matrix |
| $(u_r,u_\varphi,u_z)$ | Displacements of the shell in the radial, circumferential and axial directions |
| $\boldsymbol{V}$ | velocity of the external flow |
| $V_n$ | modal state vector |
| $W^I$ | incident power flow per unit length |
| $W^T$ | transmitted power flow per unit length |
| $(x,y,z)$ | components of global Cartesian coordinate |
| $\boldsymbol{Y}$ | state vector |
| $\nabla$ | Laplacian operator |
| $\gamma,\ \gamma_{min},\ \gamma_{max}$ | angle and critical angle of incidence |
| $\rho_3,\ \rho_1$ | density of external and internal medium |
| $\rho_c$ | mass density of shell per unit area |
| $\varepsilon_n$ | Neumann factor |
| $\varepsilon_{ij}(\gamma_{ij})$ | strain (shear strain) components |
| $\sigma_{ij}(\tau_{ij})$ | stress components |
| $\bar{\tau}$ | average power transmission coefficient |
| $\upsilon$ | Poisson's ratio |
| $\omega$ | angular frequency |
| $\eta$ | dimensionless radial coordinate |

the definition of the ring and critical frequencies to an infinite orthotropic shell excited by a plane wave. In addition, they presented a model for the acoustic transmission of oblique incidence of multi-layered cylindrical shells [7]. Finally, the same authors presented a three dimensional model which considers 3D displacement fields in the thickness for the acoustic transmission through an orthotropic multi-layered infinite cylindrical shell [8,9]. Lee and Kim [10,11] calculated the sound transmission in cylindrical shells using analytical and experimental model. The inside cavity was assumed to be anechoic and the incident wave was a plane wave. The shell vibration motions were described by the Love's equations. They considered all three displacement fields and both transverse and in-plane equations to depict the shell motion. Additionally, due to the solution procedure based on a series form, they applied a convergence algorithm in calculating TL. Eventually, they have presented the good agreement for their analytical and experimental models. Transverse shearing and rotational inertia were still neglected.

Following this researches, Tang et al. [18,19] studied sound transmission through infinite cylindrical sandwich shells illuminated an oblique plane wave with two different incident angles. They presumed same Blaise's [6] assumptions for incident angles and acoustic media. The effects of external airflow and pressure difference between inside and outside shell surfaces were considered in their work. Also, in order to calculate the TL, they utilized Naghdi–Berry theory [20] and first order theory for thick and thin shells, respectively.

In most literatures cited above, the classical shell theory (CST) was utilized to model the shell vibration and it cannot be used for thick shells and even thin shells when the number of circumferential waves increases as the result of neglecting shear deformation and rotary inertia effects in CST. In these cases, implementation of CST for thick and relatively thin shells can cause solemn errors.

Many works such as those by Qatu [21–23], Kapania [24], Noor et al. [25,26], Carrera [27], Ganapathi et al. [28], Asadi et al. [29] and Wang et al. [30] are studied about different types of dynamic analyses of thick cylindrical shells by using higher-order shell theories and also thick shell theories, including shear deformation and three-dimensional theories, shallow and deep theories, linear and non-linear theories. However, there has been little analytical investigation on sound transmission of thick cylindrical shells.

Just recently, Daneshjou et al. [12–14] obtained an exact solution in a series form based on CST and first order shear deformation theory (FSDT) by considering all three displacements of the shell for orthotropic and laminated composite cylindrical shells. They are also indicated that considering the effects of shear and rotation in FSDT for thin shells leads to a decrease of TL in high frequency range in comparison to CST. Daneshjou et al. [15] later proposed an improved model for sound transmission through relatively thick FGM cylindrical shells based on third order shear deformation theory (TSDT). In addition, they have shown that for relatively thick shells where the shear and rotation effects become more significant in lower $R/h$ ratio, TSDT presents more accurate results. The shell theory may be applied to many thin cylindrical structures, while thicker cylinders can be accurately studied only by using the three-dimensional theory of elasticity. It should be noted that the theory of elasticity can be applied to thick or thin cylindrical structures.

Cao et al. [16] derived the implicit governing equations of arbitrary angle-ply laminated cylindrical shells from the 3D higher-order shell theory and solved on the basis of the Fourier transform. They are indicated that the helical wave spectra of the higher-order radial displacements are nearly separate from those of the low-order radial displacement and play a minor role in far-field acoustic radiation, which makes the two simple shell theories applicable in prediction of acoustic power of the cylindrical shells in the much higher frequency range.

The literature clearly shows that, there has been no three-dimensional investigation on sound transmission of thick cylindrical shells. Therefore, this paper presents a novel and accurate modeling of the acoustic transmission of an orthotropic cylindrical shell with an arbitrary thickness with subsonic external flow based on the 3D exact equations of anisotropic elasticity. A laminate approximate model in conjunction with the state space formulation along with the transfer matrix approach is used. Then, the results obtained were compared with the ones available in the literature and a good agreement between them was noticed. Another objective of this study is to quantify the effects of orthotropic cylindrical shell characteristics on TL. At last, the numerical