Composite Structures 132 (2015) 310–320

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Spectral finite element for wave propagation analysis of laminated composite curved beams using classical and first order shear deformation theories

Namita Nanda, Santosh Kapuria *

Department of Applied Mechanics, Indian Institute of Technology Delhi, Hauz Khas, New Delhi 110016, India

ARTICLE INFO

Article history:
Available online 14 May 2015

Keywords:
Spectral element
Laminated composite
Curved beam
Dispersion
Wave propagation

ABSTRACT

A frequency domain spectral finite element formulation is presented for the wave propagation analysis of laminated composite curved beams using the first order shear deformation theory (FSDT) and the classical laminate theory (CLT). The elements are derived from the exact solution of the governing equation of motion in frequency domain, obtained through Fourier transformation of the time domain equation. The formulation is validated by comparing the results for natural frequencies with the published results. The new elements are then employed to perform dispersion and wave propagation analyses of curved composite beams. The numerical results reveal that the wavenumbers predicted by the CLT show large deviation from those of the FSDT even for thin beams, and the deviation increases and occurs at lower frequencies with the increase in the thickness to radius ratio. The orthotropy ratio of the composite has a significant effect on the wavenumbers for tangential and mid-surface rotation modes. The wave propagation response predicted by the CLT differs widely from the FSDT prediction, for thin and thick, and shallow and deeply curved beams at both low and high frequencies. Thus, the CLT should not be used for wave propagation analysis of even thin curved laminated beams.

1. Introduction

Laminated anisotropic composite materials are fast replacing the isotropic materials in various aerospace, automotive, civil, marine and other structures due to several advantages such as the high strength-to-weight ratio, stiffness-to-weight ratio, and their ability to be tailored for specific applications by variation of the fiber orientation and stacking sequence. Laminated composite curved beams act as lightweight load carrying structural members in many modern structures. Several studies on the vibration analysis of curved composite beams have, therefore, been reported during the past few decades, a review of which can be seen in [1]. As three dimensional (3D) elasticity solutions for beams are analytically difficult to obtain and computationally involved, one dimensional (1D) beam theories have been proposed in which the 3D elasticity equations are reduced to 1D equations, by making a priori assumptions for the variations of the displacement variables across the beam cross-section. The application of the classical laminate theory (CLT) or Euler–Bernoulli beam theory [2], where the shear deformation and rotary inertia are neglected, is restricted to very thin geometry and low frequency range. For laminated composite beams, the shear deformation and rotary inertia play a key role in the response. The first order shear deformation theory (FSDT) or the Timoshenko beam theory, accounting for the effects of shear deformation and rotary inertia, is a simple and efficient 1D model for analysing moderately thick structures [3,4]. Several higher order shear deformation theories (HSDT) [5,6] and layerwise approaches [7] have also been employed for the vibration response of curved composite beams.

Dynamic problems such as the impact and elastic wave propagation studies in laminated composite structures have received much attention in recent years, for applications in structural health monitoring (SHM), damage assessment due to blast and/or impact loading etc. The finite element method (FEM) provides a convenient tool for the analysis of structures of complex geometry and boundary conditions. But, for modeling wave propagation, the element mesh size should be typically $\frac{1}{20}$th of the smallest wavelength [8]. This increases the problem size, and the computational cost becomes prohibitively large for modeling high frequency waves used in SHM applications. Besides, the standard FEM is known to introduce numerical dispersion and dissipation errors in the

* Corresponding author. Tel.: +91 11 2659 1218; fax: +91 11 2658 1119.
E-mail addresses: nanda.namita@gmail.com (N. Nanda), kapuria@am.iitd.ac.in (S. Kapuria).

http://dx.doi.org/10.1016/j.compstruct.2015.04.061
0263-8223/

solution of wave propagation problems [9]. More recently, various modifications to the classical FEM have been proposed for the wave propagation analysis in composite structures, to overcome these shortcomings. One such method is the time domain spectral finite element method (SFEM) proposed by Patera [10]. It employs higher-order interpolation functions with specific nodal positions, such as Gauss–Lobatto–Legendre points and Chebyshev points, which reduce the mass matrix to either sparse or diagonal, making it more efficient than the conventional FEM. It has been used for analysis of guided wave propagation in composite beams, plates and panels [11–15]. The method shows reduced numerical dispersion, but the issue of large problem size remains.

Another approach is the frequency domain spectral element method (SEM), which is also often called as SFEM. In this method, proposed by Doyle [16], the dynamic problem is transformed to the frequency domain using Fourier, Wavelet or Laplace transforms. It enables one to obtain a frequency dependent dynamic stiffness matrix for an element using exact or near-exact analytical solutions. The assembled system of equations is solved in the frequency domain, and the response is transformed back to the time domain using the corresponding inverse transformation. The concept of dynamic stiffness matrix had earlier been limited only to free vibration studies [18,17,19], without undergoing the spectral analysis procedure to obtain the time domain response. The Fast Fourier Transform (FFT) based SFEM has been presented in [20–22] for wave propagation in isotropic rods and beams. Similar SFEMs for laminated composite beams have been presented using the CLT [23], FSDT [24,25], and recently, an efficient layerwise theory [26]. SFEMs employing the wavelet transformation have also been presented for composite beams and plates [27,28] based on the FSDT. The use of frequency domain SFEMs for damage detection in beam structures through wave propagation has also been reported. The analysis of wave propagation in delaminated multilayer composite beams using the FFT and wavelet based SFEMs was presented using the usual FSDT [29–32] and considering the thickness deformation as well [33].

In curved beams, the extensional and flexural modes are always coupled, which makes their response more complex. A few studies on the application of the wave propagation based approach for the free vibration analysis of isotropic curved beams have been reported [34–36]. But, even though there have been many studies on the wave propagation analysis of straight composite beams structures, studies on its curved counter part have not been reported so far in the literature. The authors [37] have recently developed an FFT based SFEM for isotropic curved beams using the classical shell theory, FSDT and a refined third order theory. In this paper, we extend this work to develop FFT based spectral finite elements for wave propagation analysis of curved composite beams using the FSDT and the CLT. The formulation incorporates linear viscous damping. The developed elements are used to obtain wave dispersion relations, free vibration response and wave propagation response under modulated sinusoidal tone burst excitation for composite beams. The results of the two models are compared to understand the effect of shear deformation on these responses.

2. Governing equations of motion

2.1. First order shear deformation theory

Consider an L-layered composite circular curved beam (Fig. 1) of thickness h, width b and radius R of the middle surface. The mid-surface is considered as the reference surface ($z = 0$). The z-coordinate of the top of the kth layer numbered from the bottom is denoted as z_k, and its material symmetry direction x_1 makes an angle α_k to the circumferential axis, θ. In the FSDT, the variations of the tangential displacement u and the radial displacement w along the thickness direction are approximated as

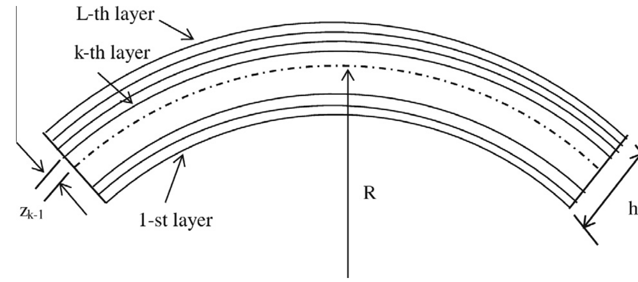

Fig. 1. Composite curved beam section.

$$u(\theta,z,t) = u_0(\theta,t) + z\psi_0(\theta,t), \quad w(\theta,z,t) = w_0(\theta,t) \tag{1}$$

where t denotes time, and u_0, w_0 and ψ_0 denote the tangential displacement, the radial displacement and the rotation of the mid-surface, respectively. The strain–displacement relations in the polar coordinate system (r,θ) are given by

$$\varepsilon_\theta = (u_{,\theta} + w)/(R+z), \quad \gamma_{z\theta} = u_{,z} + (w_{,\theta} - u)/(R+z) \tag{2}$$

where a subscript comma denotes partial differentiation. The tangential and shearing strain components, ε_θ and $\gamma_{z\theta}$, are related to the corresponding stress components σ_θ and $\tau_{z\theta}$ for the kth lamina as

$$\sigma_\theta = \hat{Q}^k_{11}\varepsilon_\theta, \quad \tau_{z\theta} = \hat{Q}^k_{55}\gamma_{z\theta} \tag{3}$$

where $\hat{Q}^k_{11}$ and $\hat{Q}^k_{55}$ are the stiffness coefficients of the kth lamina, which are obtained from the Young's moduli E_i, shear moduli G_{ij} and Poisson's ratio ν_{ij} of the kth layer as

$$\begin{aligned}&\hat{Q}_{11} = 1/\bar{s}_{11}, \quad \hat{Q}_{55} = 1/\bar{s}_{55}, \quad c = \cos\alpha_k, \quad s = \sin\alpha_k\\ &\bar{s}_{11} = c^4 s_{11} + c^2 s^2 (2s_{12} + s_{66}) + s^4 s_{22}, \quad \bar{s}_{55} = s^2 s_{44} + c^2 s_{55}, s_{11} = 1/E_1,\\ &s_{22} = 1/E_2, \quad s_{12} = -\nu_{12}/E_1, \quad s_{44} = 1/G_{23}, \quad s_{55} = 1/G_{13}, \quad s_{66} = 1/G_{12}.\end{aligned} \tag{4}$$

The equations of motion for the curved beam and the corresponding variationally consistent boundary conditions are derived from the Hamilton's principle which states

$$\int_{t_1}^{t_2} (\delta T - \delta U + \delta W)\mathrm{d}t = 0 \tag{5}$$

for all virtual displacements δu_0, δw_0 and $\delta\psi_0$ that vanish at $t = t_1,\ t_2$, where δT and δU are the first order variations of kinetic and strain energies of the beam, respectively, and δW denotes the virtual work done by the external forces. For a curved beam with span angle α, these variations are obtained as

$$\delta T = \int_0^\alpha \int_z \rho^k(\dot{u}\delta\dot{u} + \dot{w}\delta\dot{w})(R+z)b\,\mathrm{d}z\,\mathrm{d}\theta \tag{6}$$

$$\delta U = \int_0^\alpha \int_z (\sigma_\theta\delta\varepsilon_\theta + \tau_{z\theta}\delta\gamma_{z\theta})(R+z)b\,\mathrm{d}z\,\mathrm{d}\theta \tag{7}$$

$$\begin{aligned}\delta W = \int_0^\alpha &[(q_z^2(1+z_L/R) - q_z^1(1+z_0/R) - \eta_2\dot{w}_0)\delta w\\ &- \eta_1\dot{u}_0\delta u_0]R\,\mathrm{d}\theta + \langle\ \sigma_\theta\delta u + \tau_{z\theta}\delta w\rangle|_0^\alpha\end{aligned} \tag{8}$$

where ρ^k denotes the mass density of the kth layer, and q_z^1 and q_z^2 denote the normal tractions applied on the inner ($z = z_0 = -h/2$) and outer ($z = z_L = h/2$) surfaces of the beam, and η_1 and η_2 denote the viscous damping constants associated with the tangential and radial velocity components, respectively.

Substituting the expressions of displacements and strains from Eqs. (1) and (2) into Eqs. (6)–(8), performing the integration over