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An analytical model for collinear cracks in functionally graded materials with general mechanical properties

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ABSTRACT

The crack problems of functionally graded materials (FGMs) with general mechanical properties are usually to solve analytically. In this paper, an analytical model for collinear cracks in FGMs with arbitrary mechanical properties is developed. In the model, the general mechanical properties of the FGM are approached by dividing it into some layers with continuous properties along the gradient direction. The problem is finally reduced to a group of singular integral equations which are solved by numerical method. The interaction between the collinear cracks is discussed. The influences of the nonhomogeneity constants and geometric parameters on the stress intensity factors (SIFs) are analyzed. Some important conclusions are drawn.

1. Introduction

First proposed in 1980s, functionally graded materials (FGMs) have been introduced and applied in the development of structure components subject to non-uniform service requirement. For example, the application of a ceramic layer as a thermal barrier coating for a metal substrate often produced debonding at the interface after a small number of thermomechanical load cycles. FGMs, characterized by the variation in composition and structure gradually over volume, can be minimize the mismatch of material properties and improve the boding strength. Fracture failure is a key failure mode of FGMs in service, hence successful application of these materials depends on the researching and understanding of their fracture mechanics.

Fracture analysis for FGMs has received considerable attention by many researchers. Konda and Erdogan [1] considered the general mixed mode plane strain problem for an arbitrarily oriented crack in a nonhomogeneous medium. Chen and Erdogan [2] conducted the debonding problem for a composite layer that consisted of a homogeneous substrate and a nonhomogeneous coating. It was assumed that the problem was one of plane strain or generalized plane stress and the elastic medium contained a crack along the interface. Gu and Asaro [3] analyzed a semi-infinite crack in a strip of an isotropic, functionally graded material under edge loading and in-plane deformation conditions. Their results were extended to the case where the strip is made of an orthotropic, functionally graded material. Guo and Noda [4] studied the thermal SIF of an edge crack in a functionally graded plate. Zhang et al. [5] investigated the thermal shock fracture problem of FGMs. Guo et al. [6] and Guo et al. [7] developed a new interaction integral method for solving the crack problems of nonhomogeneous materials with complex interfaces. Rao and Rahman [8] presented two new interaction integrals for calculating stress intensity factors for a stationary crack in two-dimensional orthotropic functionally graded materials of arbitrary geometry. A theoretical treatment of mode I crack problem is put forward for a functionally graded orthotropic strip by Guo et al. [9] in which the singular integral equation for solving the problem and the corresponding asymptotic expression of the singular kernel are obtained. Three significant loading conditions are considered during the analysis in it. Guo and Noda [10] proposed an analytical method for a functionally graded layered structure with a crack crossing the interface and depicted the variation of the SIFs with the representative parameters when the crack moves from one layer into another layer. Dag and IIhan [11] presented analytical and computational methods for mixed-mode fracture analysis of an orthotropic FGM coating-bond coat- substrate structure. Guo et al. [12] studied the transient fracture behavior of a functionally graded coating-substrate structure. Torshizian and Kargarnovin

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[13] studied mixed-mode fracture mechanics analysis of an embedded arbitrarily oriented crack in a two-dimensional functionally graded material using plane elasticity theory. It should be mentioned that in the past decades, the mechanical properties in the analytical models for the crack problems of FGMs were usually assumed to be very particular functions so that the problems can be solved. However, this particular assumption is usually not practical. Greatly different from the previous models, Guo and Noda [14] proposed a piecewise-exponential model (PE model) to solve the crack problems of FGMs with general mechanical properties. Since the PE model is based on the analytical model of a single layer with exponential properties, it provides significant convenience for solving the crack problems of FGMs with general properties. Following this work, Guo and Noda [15] proposed an analytical thought combining the piecewise-exponential model and a perturbation method to investigate the thermal shock crack problems of FGMs with general thermomechanical properties. Guo et al. [16] developed an analytical fracture mechanics model to predict the stress intensity factors in FGMs with stochastic uncertainties in phase volume fractions. Wang et al. [17] considered the anti-plane crack problem in a functionally graded material strip. Torshizian and Kargarnovin [18] proposed an internal crack located within a FGM strip bonded with two dissimilar half-planes and under an anti-plane load.

Some authors have investigated multiple cracks in nonhomogeneous or functionally graded materials. Shbeeb et al. [19,20] dealt with the general solution to a single and multiple oriented cracks embedded in a nonhomogeneous infinite plate. They assumed that the shear modulus is of an exponential form. Vialaton et al. [21] presented a method of numerical analysis for the problem of two collinear cracks in a finite, linear elastic, isotropic plate and subjected to in plane forces which might be considered as an alternative to the body force method. Parihar and Sowdamini [22] considered the plane strain problem of determining the distribution of stress in the vicinity of three cracks embedded in an infinite isotropic elastic medium. In this problem, the cracks are collinear, the two side cracks were equal in length and located symmetrically with respect to the middle cracks. The surface tractions acting on the cracks were completely arbitrary. Tvardovsky [23] analyzed the influence of isolated collinear cracks in every other layer of a laminate with three or more layers. The material was homogeneous and anisotropic but also alternative one layer to another. Chen [24] presented elastic analysis for a collinear crack problem in antiplane elasticity of functionally graded materials. Ding and Li [25] dealt with two bonded functionally graded finite strips with two collinear cracks by using the integral transform, singular integral equation methods and the theory of residues. The bi-parameter exponential functions were introduced to express the continuous variations of the shear modulus of both FGM layers. Xu and Wu [26] presented fracture mechanics analyses for multiple collinear cracks using the weight function method and obtained stress intensity factors, crack opening displacements for three equal/unequal length collinear cracks. Chang and Kotousov [27] investigated the interaction between two collinear cracks of equal length taking into account the plate's thickness effect.

It should be mentioned that to solve the multiple cracks problem analytically, most of the above theoretical investigations assumed the material properties to be exponential functions. However, as stated above, these assumptions may not be practical for actual FGMs. In this paper, an analytical model for collinear cracks in FGMs with arbitrary mechanical properties is developed. And then, the interaction between the collinear cracks is discussed. The influences of the nonhomogeneity constants and geometric parameters on the stress intensity factors (SIFs) are discussed.

2. Problem formulation for FGMs with two collinear cracks

A functionally graded strip with arbitrarily distributed properties along the thickness direction is shown as in Fig. 1. Two internal cracks are collinearly located perpendicularly to the free surfaces. The thickness of the strip is l.

According to the principle of superposition, the solution of this boundary-value problem can be divided into two parts, as shown in Fig. 2.

The practical properties of functionally graded material are distributed randomly based on the spatial localization, which makes some mechanics problems without chance to be analytically settled. Therefore, as it is shown in Fig. 3, the progressive multi-layered model is put forward. In the novel scheme, the graded material is separated into M nonhomogeneous layers in the direction of the thickness direction, and the properties of each layer are varied exponentially. And the practical properties of material are described in virtue of a sequence of exponential functions, by which the ever-different characteristics of FGMs are approximated exactly. In the Fig. 3, the material is separated into M layers, subscript n ($n = 1, 2, \ldots, M$) is used to mark each layer, and $n_{11},\ n_{12},\ n_{21}$ and n_{22} is used to denote layer with the crack tips, where $n_{11} < n_{12} < n_{21} < n_{22}$ exists. Counting from the top surface of the structure, the nth layer is positioned between the region $x = l_{n-1}$ and $x = l_n$, where $l_0 = 0$ exists. The Young's modulus E is varied in the direction of x-axis, and the Young's modulus of the both surfaces in the functionally graded strip are expressed as E_0 and E_M, respectively.

The real shear modulus of the functionally graded strip with arbitrary properties is

$$\mu(x) = z(x) \tag{1}$$

where $z(x)$ is a known arbitrary real function. If we assume the shear modulus of each layer as

$$\mu_n(x) = \mu_{n0} e^{\delta_n x}, \quad n = 1, 2, \ldots, M \tag{2}$$

and use the real material properties on both surfaces of each layer, then we have

$$\begin{cases} \mu_n(l_{n-1}) = z(l_{n-1}) = \mu_{n0} e^{\delta_n l_{n-1}} \\ \mu_n(l_n) = z(l_n) = \mu_{n0} e^{\delta_n l_n} \end{cases}, \quad n = 1, 2, \ldots, M \tag{3}$$

From the above equations, μ_{n0} and δ_n can be solved as

$$\begin{cases} \delta_n = \frac{1}{l_n - l_{n-1}} \ln\left[\frac{z(l_n)}{z(l_{n-1})}\right] \\ \mu_{n0} = z(l_n) e^{-\delta_n l_n} \end{cases}, \quad n = 1, 2, \ldots, M \tag{4}$$

Now we will discuss the solution of this problem in detail. Assume that $\sigma_{nxx},\ \sigma_{nyy}$ and σ_{nxy} denote global stresses and u_n and v_n denote global displacements, and $\sigma_{nxx}^{(k)},\ \sigma_{nyy}^{(k)}$ and $\sigma_{nxy}^{(k)}$ denote stress components, $u_n^{(k)}$ and $v_n^{(k)}$ denote displacement components, respectively. As shown in Fig. 2, according to superposition principle, relationships between the global and the components can be expressed as

$$\left[\sigma_{nxx} \quad \sigma_{nxy} \quad \sigma_{nyy}\right] = \sum_{k=1}^{2} \left[\sigma_{nxx}^{(k)} \quad \sigma_{nxy}^{(k)} \quad \sigma_{nyy}^{(k)}\right], \quad n = 1, 2, \ldots, M \tag{5}$$

$$\left[u_n \quad v_n\right] = \sum_{k=1}^{2} \left[u_n^{(k)} \quad v_n^{(k)}\right], \quad n = 1, 2, \ldots, M \tag{6}$$