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Stochastic finite element analysis of composite structures based on material microstructure

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ABSTRACT

The linking of microstructure uncertainty with the random variation of material properties at the macroscale is particularly needed in the framework of the stochastic finite element method (SFEM) where arbitrary assumptions are usually made regarding the probability distribution and correlation structure of the macroscopic mechanical properties. This linking can be accomplished in an efficient manner by exploiting the excellent synergy of the extended finite element method (XFEM) and Monte Carlo simulation (MCS) for the computation of the effective properties of random two-phase composites. The homogenization is based on Hill's energy condition and involves the generation of a large number of random realizations of the microstructure geometry based on a given volume fraction of the inclusions and other parameters (shape, number, spatial distribution and orientation). In this paper, the mean value, coefficient of variation and probability distribution of the effective elastic modulus and Poisson ratio are computed taking into account the material microstructure. The effective properties are used in the framework of SFEM to obtain the response of a composite structure and it is shown that the response variability can be significantly affected by the random microstructure.

1. Introduction

A powerful tool in computational stochastic mechanics is the stochastic finite element method (SFEM). SFEM is an extension of the classical deterministic FE approach to the stochastic framework i.e. to the solution of stochastic problems whose (material and geometric) properties are random with the FE method. The considerable attention that SFEM received over the last two decades can be mainly attributed to the understanding of the significant influence of the inherent uncertainties on systems behavior and to the dramatic increase of the computational power in recent years, rendering possible the efficient treatment of large-scale problems with uncertainties [1]. In most SFEM applications, the description of macroscopic material properties using random variables/fields is based on arbitrary assumptions of the respective probability distribution. The simplest option consists of a straightforward randomization of one or more material parameters, used in deterministic constitutive models. In the case of a linear elastic analysis, only the Young modulus is typically assumed to be random [2,3], but examples where the Poisson ratio is treated as a random field are available, e.g. [4–6]. In addition, the correlation structure of the random field is often arbitrarily assumed and the correlations of different material parameters are typically ignored in the random field model. This is mainly due to the fact that insufficient experimental evidence is usually available to validate all of the detailed characteristics of a macro-random field.

The mechanical behavior of heterogeneous and in particular of composite materials is governed by the mechanical properties of their individual components, their volume fractions and other parameters defining their spatial and size distribution. As mentioned before, only the macroscopic mechanical behavior is of interest in many cases. However, the microstructure attributes of this type of materials are extremely important for a better understanding of their intrinsic properties. This is the reason for which the linking of micromechanical characteristics with the random variation of material properties at the macroscale has gained particular attention in recent years. In [7], the quantitative characterization of the microstructure of random heterogeneous materials is treated in detail and the connection between material properties and microstructure is established for several cases. In [8], a process for the evaluation of stochastic formulations for modeling the constitutive behavior of heterogeneous solid materials is proposed. A FEM incorporating microstructural material randomness below

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the level of a single (mesoscale) finite element is described in [9]. A moving-window micromechanics technique is applied in [10] to produce material property fields associated with the random microstructure of particulate reinforced composites. A homogenization procedure for determining effective properties of composite structures with stochastic material characteristics is considered in [11]. Auto- and cross-correlations of material properties are estimated in [12] using simple micromechanical models and homogenization. The relation between various microstructural properties and the overall behavior of stochastically heterogeneous beams is studied in [13]. The effect of microstructural randomness on the fracture behavior of composite cantilever beams is investigated in [14] through the application of cohesive zone elements with random properties. The generalized variability response function (GVRF) methodology is used in [15] to compute the displacement response and the effective compliance of linear plane stress systems. In [16], the macroscale response of polycrystalline microstructures and the accuracy of homogenization theory for upscaling the microscale response are examined by performing direct numerical simulations of this type of microstructures embedded throughout a macroscale structure (I-beam). The influence of a microscopic random variation of the elastic properties of component materials on the mechanical properties and stochastic response of laminated composite plates is investigated in [17].

Despite the aforementioned efforts, the number of publications highlighting the role of microstructure uncertainty in the framework of SFEM is still very limited. As a step forward in this direction, the effective properties of random two-phase media can be computed by exploiting the excellent synergy of the extended finite element method (XFEM) and Monte Carlo simulation (MCS). The homogenization is based on Hill's energy condition and involves the generation of a large number of random realizations of the microstructure geometry based on a given volume fraction of the inclusions and other parameters, such as shape, number, spatial distribution and orientation (Section 2). The statistical characteristics (mean value, coefficient of variation and probability distribution) of the effective elastic modulus and Poisson ratio are computed taking into account the material microstructure (Section 3). In Section 4, the effective properties are used in the framework of SFEM to obtain the response of a composite structure and useful conclusions are derived regarding the effect of random microstructure geometry on the probabilistic characteristics of the response.

2. Homogenization of random heterogeneous media using XFEM and MCS

The linking of micromechanical characteristics with the random variation of material properties at the macroscale has become a prerequisite in material characterization, especially in nanocomposites [18]. Specifically, the mechanical behavior of such heterogeneous materials is governed by the mechanical properties of their individual components, their volume fractions and other parameters defining their spatial and size distribution. In a previous work by Savvas et al. [19], it is highlighted that the statistical characteristics of the effective properties can also be significantly affected by the shape of the inclusions, especially in the case of large volume fraction and stiffness ratio. For example, this is the case of graphene nano-platelet (GnP)-reinforced composites where the stiffness ratio between the polymer matrix and graphene nanofiller is approximately 1/1000.

Effective homogeneous properties of composite materials can be extracted analytically, numerically or in a hybrid manner through appropriate homogenization techniques, e.g. [7,11,12,17,20–27]. A novel homogenization approach was presented in [19], where extended finite element analysis of microstructure coupled with MCS was used. This MCS based stochastic homogenization approach involves the computational analysis of a large number of randomly generated realizations of the composite medium using XFEM. The results derived from the micromechanical analysis are then used in the calculation of the effective properties of an equivalent homogeneous medium, where Hill's energy condition is satisfied. These effective properties will serve as a basis for the stochastic finite element analysis of structures made from random composites, as explained in Section 3.

It is worth noting that in [19], the influence of inclusion shape on the effective Young modulus and Poisson ratio of the homogenized medium was demonstrated through histograms of the corresponding properties, along with the statistical convergence of their mean and coefficient of variation (COV). In Section 2.2 of this paper, the effect of inclusion shape on the effective properties of the random composites will also be quantified in terms of the inclusion surface to volume ratio with regard to the mean values of the stochastic material properties for various volume fractions.

2.1. Methodology

2.1.1. Problem formulation

Consider a medium which occupies a domain $\Omega \subset \mathbb{R}^2$ whose boundary is represented by Γ. Let prescribed traction $\bar{t}$ applied on surface $\Gamma_t \subset \Gamma$ (natural boundary conditions) and prescribed displacements $\bar{u}$ applied on $\Gamma_u \subset \Gamma$ (essential boundary conditions). The medium contains an inclusion which occupies the domain Ω^+ and is surrounded by the internal surface $\Gamma_{incl} \subset \Gamma$ such that $\Omega = \Omega^+ \cup \Omega^-$ and $\Gamma = \Gamma_t \cup \Gamma_u \cup \Gamma_{incl}$ (Fig. 1). The governing equilibrium and kinematic equations for the elastostatic problem of the medium is:

$$\text{div}\,\boldsymbol{\sigma} + \boldsymbol{b} = 0 \quad \text{in } \Omega \tag{1}$$

$$\boldsymbol{\sigma} = \mathbb{C} : \boldsymbol{\varepsilon} \tag{2}$$

where $\boldsymbol{b}$ are the body forces acting on the medium, $\boldsymbol{\varepsilon} = \frac{1}{2}(\nabla u + \nabla u^T)$ is the second order tensor of the measured strains and $\mathbb{C}$ is the fourth order elasticity tensor. The essential and natural boundary conditions are:

$$u = \bar{u} \quad \text{in} \quad \Gamma_u \tag{3}$$

$$\boldsymbol{\sigma} \cdot n = \bar{t} \quad \text{in} \quad \Gamma_t \tag{4}$$

where n is the unit outward normal to Γ_t.

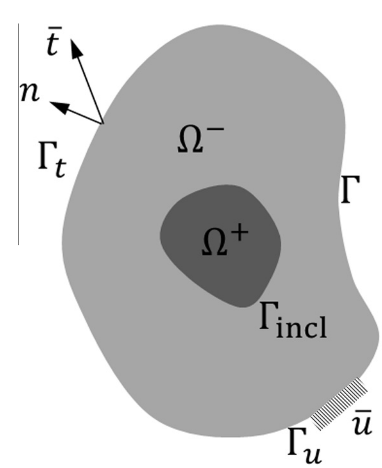

Fig. 1. Schematic of a medium which occupies a domain $\Omega = \Omega^+ \cup \Omega^-$, contains an inclusion (Ω^+) and is subjected to essential and natural boundary conditions on surfaces Γ_u and Γ_t respectively.