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Continuous inter-laminar stresses for regular and inverse geometrically non linear dynamic and static analyses of laminated plates and shells

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ABSTRACT

This study presents an orthotropic laminated finite element with continuous stress distribution along transverse direction applied to geometrically non linear analysis of static and dynamic plates and shells. The kinematic description is total Lagrangian based on positions and generalized vectors which avoids the use of the finite rotation concept. Therefore, the loss of precision that may appears when reference updating takes place is not present. Moreover, there is no necessity to transform velocity and acceleration from the spatial to the material reference frame and the Newmark method can be applied.

Using generalized vectors introduces constant thickness variation and complete 3D constitutive relation. Without increasing the number of degrees of freedom, the equilibrium of laminas parallel to the reference surface is applied to achieve continuous stress distribution along transverse direction. A curved triangular finite element with cubic approximation is adopted also avoiding membrane locking.

Static examples are designed to check the absence of locking and to verify the quality of transverse stress distribution for thin and moderately thick orthotropic plates and shells. Dynamic examples show the ability of total Lagrangian formulations on solving general and inverse dynamic problems with energy conservation.

1. Introduction

Highly deformable structures are found in various engineering applications, as deployable structures used in aerospace industry and in medicine. Moreover, current thin or not thin walled structures may also suffer large displacements and rotations, as for example, metal profiles in civil, aeronautical and mechanical engineering, hulls of submarines in naval industry, tubes for liquid or gas transportation in chemical industry etc. Laminated plates and shells are usually applied to achieve the desired stiffness and lightness for parts of these structures.

One of the most spread techniques to model this kind of structural elements, subject or not to large displacements, is the finite element method (FEM). Considering the occurrence of large displacements the updated Lagrangian FEM formulations, including the ones that consider large displacements and rotations with small strains (co-rotational) are the most adopted, see some reference works as [1–4]. However, as described by various authors [5–7], in addition to accumulate errors due to the inexact displacement–strain relation at the updating process, most updated Lagrangian techniques adopts finite rotation formulas limiting the load steps for general applications and preventing the solution of inverse problems. One should not forget that, in updated Lagrangian formulations velocity and acceleration should be transformed from spatial to material reference frame by formulas associated with the finite rotation. This process should be satisfactorily treated using specially developed time integrators, see references [8–13]. As total Lagrangian formulations for the dynamic analysis of structures are practically inexistent, most authors of updated Lagrangian formulations, mainly co-rotational, indicate that the Newmark algorithm cannot be used for non-linear dynamic analysis. However, as enlightened by Pai et al. [5], Kane et al. [14], Coda and Paccola [15,16] and Sanches and Coda [17] the Newmark algorithm can be used in non linear dynamic formulations based on total Lagrangian descriptions because the adopted reference is the material one.

So, as commented by Pai et al. [5] when one needs to develop long term analysis that involves an enormous number of reference updating with not so small strains, or, when one needs to obtain inverse information to preview the storage position of deployable structures; it is of interest to adopt total Lagrangian FEM formulations.

Related to laminated plates and shells modeling, the precise calculation of stresses in the transverse direction is still a challenge

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for nowadays researchers. One can see, for example, some works that show the evolution in this field [18–23]. Despite the fact that, in its majority, the mentioned formulations follows rotation based and updated Lagrangian descriptions, the work [18] shows an excellent summary about this subject and a detailed review can be seen in [24,25]. Following [18,26] formulations for which the displacement field is continuously differentiable along the transverse direction are unable to accurately calculate transverse stress components for laminated plates and shells. The layer-wise formulation [27–29] considerably improves the zig-zag behavior of the transverse displacement; however, as mentioned by Vu-Quoc and Tan [18] and Basar et al. [30] it gives discontinuous results for stresses that can be improved if the number of degrees of freedom is greatly increased. A lot of non-conventional formulations as [31,32] use Hermite's polynomials associated to additional restrictions to guaranty the continuity of transverse stresses; however, various mathematical difficulties are found to generalize these formulations. Other techniques can be cited [33–35] but their generalization are difficult and some inconsistent results are still present [18]. A work that makes an important comparison among various classical laminated formulations is [36].

Hybrid formulations can be cited [18,37–42]; however, its development are in some sense cumbersome and are still limited to linear applications. Moreover, the number of degrees of freedom increases as the number of different laminas and material increases.

The present study introduces an orthotropic laminated finite element with continuous stress distribution along transverse direction without increasing the number of degrees of freedom. The proposed formulation is applied to geometrically non linear analysis of static and dynamic plates and shells. The kinematic description is total Lagrangian based on positions and generalized vectors avoiding the use of the finite rotation concept and correlated formulas. Therefore, as described by Pai et al. [5] and Pai [6] the loss of precision that may appears when a large number of reference updating takes place is avoided. Moreover, as the reference configuration is not updated there is no necessity to transform velocity and acceleration from the current position (spatial) to the static reference frame (material) and the Newmark method can be used to integrate time [5,14–17].

Defining the shell kinematics by generalized vectors naturally introduces a constant thickness variation; therefore, a complete 3D constitutive relation is applied. In the kinematics definition there is also present an enhancement developed for homogeneous shell analysis that imposes linear strain variation along transverse direction [43,44]. To achieve the new proposed continuous stress distribution along transverse direction without increasing the number of degrees of freedom we impose the equilibrium of laminas parallel to the reference surface, associated to the Cauchy Theorem These features makes difficult to classify the proposed formulation in the above mentioned state of art. The simultaneous action of the previous and the new enrichments eliminates the shear and volumetric locking. A curved triangular finite element with cubic approximation is adopted also avoiding membrane locking.

The proposed formulation description is as follows. Section 2 describes the positional basic kinematics of laminated plates and shells including linear thickness variation. In Section 3 the new strain enhancement that results in continuous transverse stresses for laminated orthotropic plates and shells is presented. Section 4 shows the operational application of the Newmark time integrator associated to the Newton–Raphson solution process and the angular and linear momentum conservation expressions for total Lagrangian formulations. Section 5 presents static results covering locking problems and stresses profiles. Section 6 presents dynamic examples including long term analyses and inverse problems for isotropic and laminated orthotropic plates and shells.

2. Kinematics and specific strain energy

This section is divided into two subsections in which, using unconstrained vector description, the pure and basic kinematics of laminated plates and shells are presented. The pure kinematics can be compared to the first order kinematics of usual displacement-rotation-based formulations for which an orthogonal and straight line remains straight but not orthogonal to the reference surface of the plate or shell after deformation. The main difference is the natural constant transverse stretching that occurs due to the unconstrained vector approach, a solid-like behavior.

The basic kinematics can be compared to the so called second order theory, for which a quadratic enhancement is applied along the transverse direction. However, due to the unconstrained vector approach, all directions are naturally enhanced, parallel and transverse to the reference surface. This kinematics, together with the triangular cubic approximation of variables, is sufficient to avoid shear, volumetric and membrane locking for homogeneous plates and shells [43–45].

2.1. Pure kinematics – homogeneous isotropic

As mentioned in the introduction of Section 2 the pure kinematics for a homogeneous shell (or plate) can be described by Fig. 1.

In Fig. 1 a generic point $\boldsymbol{x}$ at the initial configuration is written as a function of a point $\boldsymbol{x}^m$ at the reference surface and a generic vector $\boldsymbol{g}^0$. Using the same procedure a generic point $\boldsymbol{y}$ at the current configuration is written as a function of a point $\boldsymbol{y}^m$ at the reference surface and a generic vector (unconstrained) $\boldsymbol{g}^1$. These variables are written as function of non-dimensional coordinates (ξ_i) generating the initial $(\boldsymbol{f}^0)$ and current $(\boldsymbol{f}^1)$ mappings, as well as their corresponding gradients ($\boldsymbol{A}^0$ and $\boldsymbol{A}^1$), written as:

$$f_i^0 = x_i = \phi_k(\xi_1, \xi_2) X_{ki} + \frac{h_0}{2} \xi_3 \phi_\ell(\xi_1, \xi_2) G_{\ell i}^0 \tag{1}$$

$$f_i^1 = y_i = \phi_k(\xi_1, \xi_2) Y_{ki} + \frac{h_0}{2} \xi_3 \phi_\ell(\xi_1, \xi_2) G_{\ell i} \tag{2}$$

$$g_i^0 = \frac{h_0}{2} \xi_3 \phi_\ell(\xi_1, \xi_2) G_{\ell i}^0 \tag{3}$$

$$g_i^1 = \frac{h_0}{2} \xi_3 \phi_\ell(\xi_1, \xi_2) G_{\ell i} \tag{4}$$

$$A_{ij}^0 = \frac{\partial f_i^0}{\partial \xi_j} = f_{i,j}^0, \quad A_{ij}^1 = \frac{\partial f_i^1}{\partial \xi_j} = f_{i,j}^1 \tag{5}$$

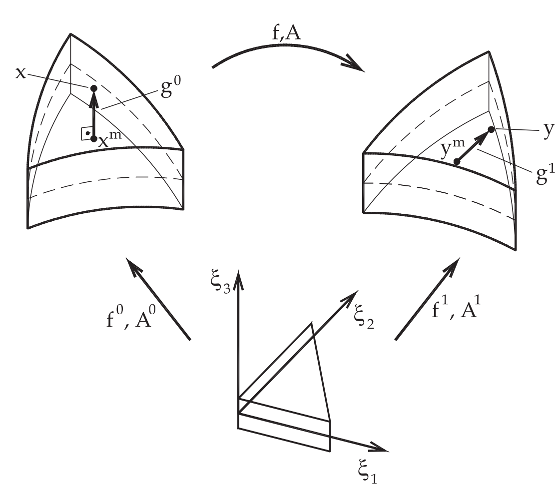

Fig. 1. Position vectors and auxiliary mappings.