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Influence of out-of-plane ply waviness on elastic properties of composite laminates under uniaxial loading

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ABSTRACT

In this paper the influence of out-of-plane ply waviness on elastic properties of composite laminates subjected to uniaxial loading is investigated. A three-dimensional analytical method based on the classical lamination theory is developed to quantitatively determine the effective elastic constants of thick composite laminates with ply waviness. Various influential factors associated with ply waviness, such as the waviness geometry, the off-axis angle, the extent of wavy region, and the waviness pattern, are incorporated in the multi-parameter model. Analytical results for the Young's moduli, the shear moduli and the Poisson's ratios of wavy fiber composite laminates are obtained and discussed for a specific range of fiber volume fractions and two different lay-ups. Negative Poisson's ratios are found for unidirectional laminates nested wavy ply/plies. The present method provides a useful and effective tool to evaluate the effects of out-of-plane ply waviness on three-dimensional elastic properties of composite laminates.

1. Introduction

Nowadays typical composite materials constituted of polymer matrix and reinforcing fibers are gaining a widespread application in many industrial fields, such as aerospace, marine, automobile, nuclear, and gas transportation engineering, due to their high specific strength and stiffness coupled with cost effectiveness compared with traditional metal materials. However, at the micro-level, fiber itself has some intrinsic imperfections with possible implications for in-service performance of composites. For lamina at meso-level and laminates at macro-level, several defects are unintentionally introduced during the process of manufacturing and are inevitably accumulated in service. Partial defects may be reduced and eliminated by improvements in manufacturing processes, but it is not practicable to eliminate the inherent defects in composite structural components. To date, a number of studies have shown that the process-induced defects have greatly adverse influence on the mechanical performance of fiber reinforced composites [1–3]. In addition, the requirements of quality control for composite products become rigorous. Thus it necessitates the development of analytic and numerical models for quantitatively determining the effects of defects on the mechanical behaviors of composite structures.

Waviness defect, or fiber misalignment, is one of the most frequently encountered defects in large, thick composite structures. It is an important source of geometric non-uniformity and material non-uniformity of composite materials, leading to nonlinearity of material behaviors and complexity of composites structural analysis. Generally, the waviness is caused by the axial compression in the fibers due to the non-uniform pressure distribution among layers as well as the difference of thermal expansion coefficients among fiber, matrix and tooling materials. It is usually categorized into two types: in-plane and out-of-plane waviness. In-plane waviness, or fiber waviness, occurring mostly during the filament-winding process, describes the fibers deviation from the nominal fiber direction in the plane of the lamina [4–8]. Out-of-plane waviness, or ply waviness, involves the cooperative undulation of a layer or multiple layers in the through-thickness direction of cross-ply, unidirectional, and multidirectional laminates [4,5,9,10]. Some researchers have demonstrated that influence of in-plane waviness is more serious than out-of-plane waviness [1,11,12]. The two waviness patterns occurred within composite materials have been frequently modeled as sinusoidal waveforms, and a Gaussian function has been also employed to simulate the waviness geometric morphology [13].

Extensive investigations have been conducted on the impact of waviness on mechanical performance of composite structures [1,7,9,10,14–16], and some representatives are briefly noted as follows. Hsiao and Daniel proposed two types of analytical methods

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for predicting the mechanical performance of composites embedded ply waviness. With the aid of the classical lamination theory to thin laminates, an analytical model [14,17] was developed to study the effects of three types of out-of-plane waviness on stiffness and strength reduction of unidirectional composites under compressive loading; they [17] further determined the elastic properties and compressive strength of cross-ply composites as a function of out-of-plane waviness. By using complementary strain energy [18], the mathematical form of the constitutive equation for the three waviness patterns was derived, and nonlinear elastic behaviors of unidirectional composites with ply waviness under compressive loading were predicted. Moreover, in their presentations several experimental results were obtained by compression tests and compared with theoretical predictions. Analytical results and measured values showed reasonable agreement.

Altmann et al. [10] enhanced Hsiao and Daniel's approach to predict the effects of matrix systems on the strength of unidirectional laminates containing ply waviness under compressive loading. With the use of the PUCK failure criterion and the software MATLAB, they constructed a graphical user interface, which was validated against the existing literature data. Their analysis indicated that failure behavior of wavy laminates is mostly affected by matrix dominated shear strength.

Chun et al. [15] developed an analytical model by applying complementary energy density and an incremental method to unidirectional composites encompassing out-of-plane waviness. In their model, material and geometric nonlinearities triggered by out-of-plane waviness were considered. Effects of the nonlinearities on the material behaviors of unidirectional laminates under tensile and compressive loadings were systematically investigated. Some tension and compression tests were then conducted on normal and wavy specimens and partial analytical predictions were validated with the experimental date. On the basis of two-dimensional and three-dimensional classical lamination theory, Bogetti et al. [19,20] proposed an analytical model to evaluate the reduction in stiffness and strength of laminates with out-of-plane waviness under tensile loading; however, they made no attempt to obtain the explicit form of this model.

Chan and Wang [21] studied influence of in-plane waviness on stiffness and its sensibilities due to variation of waviness for a composite beam. Furthermore, the explicit form of stiffness of lamina and laminate with a single-ply in-plane waviness was formulated. Lemanski and Sutcliffe [22] took into account the impact of length, width and position of in-plane waviness zone on the compressive behaviors of unidirectional composites via FEA, and a simple empirical equation was proposed to obtain the numerical results.

Garnich and Karami [23] developed FE models and carried out a linear elastic analysis to determine the stresses and strains of composite lamina for global as well as for various conditions of localized fiber waviness. Eight types of waviness commonly used in literature are taken into account in their models, which cover in-plane and out-of-plane waviness as well as uniform, graded and localized waviness.

As aforementioned, most efforts have primarily concentrated on either the elastic behaviors of wavy fiber lamina or two dimensional stiffness of thin laminates nested waviness, but overlooked the effects of waviness on the three-dimensional elastic properties of composite laminates.

The main objective of this paper is to establish a multi-parameter model for thick composite laminates containing wavy ply/plies, and to present a three-dimensional analytical method based on the classical lamination theory (CLT) for investigating the effects of out-of-plane ply waviness on elastic properties of thick laminates under uniaxial loading. The analytical model takes into account several waviness parameters of the waviness geometry (i.e. wave amplitude to wavelength ratio, a/λ), the off-axis angle (θ), the extent of wavy region (i.e. the volume percentage of the wavy region, V_w), the fiber volume fraction (V_f), and the waviness pattern (uniform or graded waviness). Two types of laminates with different lay-ups (unidirectional and symmetrical cross-ply) are taken into consideration and simulated, respectively.

2. Analytical model and methods

In order to determine the localized elastic properties of composite laminates nested wavy plies, a mathematical description for the wavy ply is required. As previous investigations, a sine or cosine function is commonly used to depict the ply waviness geometry for the considered wavy morphology, as can be seen in Fig. 1. The most important geometric parameters for the sinusoidal waveform are the wavelength or the period, λ, the amplitude, a, and the off-axis angle, θ, where the peak-to-peak amplitude of such waveform is equal to $2a$. It is assumed that out-of-plane ply waviness discussed in this work undulates in the X–Z plane and extends in the X-, Y-, and Z-direction, where X is the longitudinal direction parallel to the desired fiber direction, Y is the transverse directions normal to the X–Z plane and Z is the through-thickness direction in the global coordinate system. The morphology of the cosine waveform and three geometrical parameters (a, λ, and θ) are schematized in Fig. 1(b).

Additionally, in the X- and Z-direction, the wavy region is viewed as an assembly of many infinitesimally thin pieces, $dxdz$. Each piece is treated as a unidirectional lamina orientating at the off-axis angle, θ, relative to the nominal fiber direction. Hence the stiffness of each piece can be determined with the stress or strain transformation relation. Based on this and CLT, the three-dimensional elastic properties of composite laminates with ply waviness are then to be evaluated.

In the subsequent analytical model, the schematics for the uniform and graded waviness are graphically outlined, as the Model (1-2-3) for uniform waviness and the Model (3-1) for graded waviness in Reference [23]. Also, two different lay-ups of the unidirectional and symmetrical cross-ply laminates are deliberately modeled, respectively.

2.1. Description of waviness morphology

The uniform waviness (UW) represents an idealization that all fibers undulate with the identical sine or cosine waveform and are parallel to each other within the wavy region.

In the global coordinate system, the undulation form for the median wavy ply of waviness zone along the X-direction or the nominal fiber direction is expressed by $z_u(x)$,

$$z_u(x) = a\cos(2\pi x/\lambda), \quad -\lambda/2 \leqslant x \leqslant \lambda/2 \tag{1}$$

where the subscript 'u' signifies the uniform waviness.

The off-axis angle for UW is then obtained by the derivation of $z_u(x)$,

$$\tan\theta_u(x) = -\beta\sin(2\pi x/\lambda), \quad -\lambda/2 \leqslant x \leqslant \lambda/2 \tag{2}$$

Here, $\beta = 2\pi a/\lambda$ is defined.The maximum off-axis angle of uniform waviness, $\theta_{u,\max}$, generates at $x = \pm\lambda/4, \pm3\lambda/4, \pm5\lambda/4, \ldots$, namely, at the deflection point between the trough and adjacent crest of the sinusoidal waveform, as can be seen in Fig. 1(b).

For conciseness and clarity, only the case of the acute angle for the off-axis angle is taken into account, namely, $0 < \theta < \pi/2$. Consequently, $\theta_{u,\max}$ satisfies the relation,

$$\tan\theta_{u,\max} = \beta \tag{3}$$