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Guided wave propagation and mode differentiation in the layered magneto-electro-elastic hollow cylinder

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ABSTRACT

In this paper, the dispersion properties of elastic waves in the layered piezoelectric/piezomagnetic (PPC) hollow cylinder are investigated. The axisymmetric formulation is based on the Scaled Boundary Finite Element method. High-order elements are utilized for the discretization. We develop the mode sorting method to distinguish modes based on the reciprocity relation of the magneto-electro-elastic media. The dispersion relationship and wave structures of various layered PPC hollow cylinders are presented. Besides, we discuss impacts of the stacking sequence and radius-thickness ratio on the dispersive behavior and discover the frequency deletion of longitudinal modes.

1. Introduction

Piezoelectric–piezomagnetic composites (PPC) have recently attracted considerable of studies, compared to the one of single phase material. These materials are employed now in a variety of mechanical, civil and aerospace applications as sensors, actuators, and storage devices [1,2] at various scales. The growth of such applications requires the accurate knowledge of elastic wave propagation to help to sign and optimize PPC devices. Tiersten [3] derived the linear piezoelectric equations of the infinite anisotropic plate. By applying Fourier transform method, Paul and Venkatesan [4] discussed the axial waves dispersion relationship in a hollow piezoelectric cylinders. Steeart and Young [5] discovered the cut-off frequencies on wave propagation of electro-elastic composite plate. For the analysis of elastic waves on multilayered composite structures, the most common method is the matrix formalism. Wang and Rokhlin [6] obtained an asymptotic solution of a general anisotropic piezoelectric thin layer by using the transfer matrix based on a simple second-order asymptotic expansion. Reinhardt et al. [7] developed a numerically stable scattering matrix model of metal and fluid layers and half spaces and analyzed the problem of plane wave propagation in piezoelectric and dielectric multilayered. The surface impedance matrices was employed to solve the dispersion curves on wave propagation in high impedance-contrast layered plates by Zhang et al. [8]. Lahmer et al. investigated the detection of flaws [9] and dynamic fracture [10] in piezoelectric structures using the extended finite element method. However, the solution for this method may be difficult, and part of modes may be missed.

Maradudin [11] firstly presented the orthogonal function method to study wave propagation in homogeneous semi-infinite wedges. This approach based on polynomial expansions allows an algebraic eigenvalue equation to take the place of transcendental dispersion equation, to overcome previous drawbacks. Yu et al. proposed the dispersion relationships of inhomogeneous piezomagnetic-piezoelectric spherical curved plates [12] and rectangular bar [13], by using Legendre orthogonal polynomial series expansion. Mater et al. [14] presented propagation constants and wave structures in multilayered magneto-electro-elastic composite, potentially deposited on a substrate by employing the Legendre and Laguerre polynomial method. However, the orthogonal function method is only applicable to simple cross section where the boundary conditions are easily shown, and obtain the dissymmetric eigenmatrix which is disadvantage of the solution of the dispersion curves.

Compared to the above method, the finite element method has very strong adaptability in the boundary conditions and developed squares symmetric stiffness. The semi-analytical finite element (SAFE) method as a prevalent theory is exploited to investigate the guided waves. Corrtes [15,16] presented resonance behavior of an array of piezoelectric plates and obtained the dispersion relationship of elastic guided waves in piezoelectric infinite plates with inversion layers.

In this paper, the Scaled Boundary Finite Element method (SBFEM) that the scale parameter is equal to one is adopted to analyze guided wave propagation in the layered PPC hollow cylinder,

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similar to the SAFE method. The SBFEM as a novel semi-analytical approach was originally developed by Wolf and Song [17] to compute the dynamic stiffness of an unbounded domain. It is an extension of Han's work, in which dispersion problem of helical waveguide has been addressed. Gravenkamp [18] proposed the dispersion curves for axisymemetric waveguides of elastic media using the SBFEM. More, the SBFEM was extended to a mixed-mode crack propagation model based on linear elastic fracture mechanics [19]. This approach reduces a three-dimensional problem of waveguides to a one-dimensional for axisymemetric waveguides, so that only the radical length of the cross section is needed to mesh.

We give the axisymemetric formulation of the SBFEM based on the order elements. For dispersion scatters obtained from the SBFEM are difficult to differentiate from each other, a mode sorting method for axisymemetric waveguides is established to distinguish modes by applying the reciprocity relation of the magneto-electro-elastic media. Finally, We give the dispersion curves and wave structures of various layered PPC hollow cylinder and discuss impacts of the stacking sequence and radius-thickness ratio. In addition, we also discover a new phenomenon: the frequency deletion of longitudinal modes for the hollow cylindrical piezomagnetic waveguide.

2. Formulation of the SBEFM

Consider an anisotropic magneto-electro-elastic cylinder which is infinite in the z direction of wave propagation but finite in the vertical direction of the cross-section, as shown in Fig. 1. Such a material, the generalized constitutive equations are given by

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{kij}E_k - q_{ijk}H_k \tag{1a}$$

$$D_i = e_{ikl}\varepsilon_{kl} + \upsilon_{ik}E_k \tag{1b}$$

$$B_i = q_{ijk}\varepsilon_{kl} + \mu_{ik}H_k \tag{1c}$$

where $\sigma_{ij},\ D_i$ and B_i are the stress tensor, the electric displacement and magnetic induction, respectively; $\varepsilon_{kl},\ E_k$ and q_{ijk} are the strain tensor, the electric field and magnetic field, respectively; $C_{ijkl},\ e_{kij},\ q_{ijk},\ \upsilon_{ik},\ \mu_{ik}$ are material parameters, i.e., the elastic constants, the piezoelectric constants, the piezomagnetic constants, the dielectric permittivity constants and the magnetic permeability, respectively; Here, Einstein summation convention are used, where $i,\ j,\ k$ and $l = 1, 2, 3$, corresponding to $r,\ \theta,\ z$, respectively.

We define the generalized stress vector $\bar{\boldsymbol{\sigma}}$ and strain vector $\bar{\boldsymbol{\varepsilon}}$. The generalized constitutive equations can be rewritten as

$$\bar{\boldsymbol{\sigma}} = \boldsymbol{H}\bar{\boldsymbol{\varepsilon}} \tag{2}$$

where the generalized constitutive matrices

$$\boldsymbol{H} = \begin{bmatrix} C & -e^T & -q^T \\ e & \upsilon & 0 \\ q & 0 & \mu \end{bmatrix} \quad \boldsymbol{H}^* = \begin{bmatrix} C & -e^T & -q^T \\ -e & -\upsilon & 0 \\ -q & 0 & -\mu \end{bmatrix} \tag{3}$$

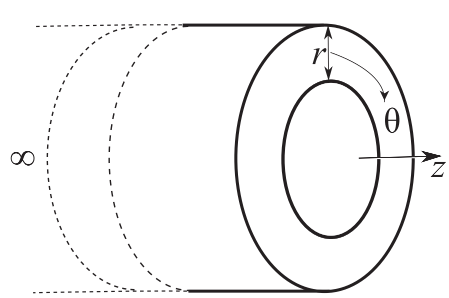

Fig. 1. Sketch map of a hollow cylinder.

The relationship between the generalized strain and generalized displacement can be expressed as

$$\bar{\boldsymbol{\varepsilon}} = \boldsymbol{L_z}\bar{\boldsymbol{u}}_{,z} + \frac{1}{r}\boldsymbol{L_\theta}\bar{\boldsymbol{u}}_{,\theta} + \boldsymbol{L_r}\bar{\boldsymbol{u}}_{,r} + \frac{1}{r}\boldsymbol{L_0}\bar{\boldsymbol{u}} \tag{4}$$

where the generalized displacement $\bar{\boldsymbol{u}} = [u_1 \quad u_2 \quad u_3 \quad -\phi \quad -\varphi]^T$, which consists of the displacements u_i, the electric potential ϕ and the magnetic potential φ; The differential operator $\boldsymbol{L}_z,\ \boldsymbol{L_\theta},\ \boldsymbol{L}_r$ and $\boldsymbol{L}_0$ in cylinder coordinates can be given in Appendix.

In this paper, the formulation of the dispersion relationship is based on the SBEFM. We can assume its displacement is independent of the time harmonic $e^{-i\omega t}$, where t is the time and ω denotes the angular frequency. The displacement after the separation of variables can be written as the Fourier series of the θ direction

$$\bar{\boldsymbol{u}} = \left(\sum_{n=0}^{\infty}\bar{\boldsymbol{u}}_r e^{in\theta}\right)e^{-i(\omega t - kz)} \tag{5}$$

There is no external forces, body forces, electric charge and current density for studying properties of propagation modes. Traction-free and open-circuit boundary conditions are assumed in this analysis:

$$\sigma_{11} = \sigma_{12} = \sigma_{13} = 0, D_1 = B_1 = 0, \quad \text{at} \quad r = r_0 \quad \text{and} \quad r_1 \tag{6}$$

where r_0 and r_1 are inside and outside radius of the hollow cylinder, respectively. Based on the Hamilton principle, the vibrational formulation of electro-magneto-elastic dynamics can be obtained as

$$\delta\int(\boldsymbol{KE} - \boldsymbol{P})dt = 0 \tag{7}$$

where $\boldsymbol{KE}$ and $\boldsymbol{P}$ are the kinetic energy and electrical enthalpy.

$$\boldsymbol{KE} = \frac{1}{2}\bar{\boldsymbol{\varepsilon}}\boldsymbol{H}^*\bar{\boldsymbol{\varepsilon}}, \quad \boldsymbol{P} = \frac{1}{2}\boldsymbol{\rho}\left(\ddot{\bar{\boldsymbol{u}}}\right)^2 \tag{8}$$

where the material density matrix $\boldsymbol{\rho}$ can be given Appendix; the subscript of $\ddot{\bar{\boldsymbol{u}}}$ is the second derivative of time.

The free boundary is founded for any θ, or can be express as the form which is independent of θ. We can adopt Eq. (5) into Eq. (7) to derive a sires of equations, exclusive of θ, based on the orthogonality of the trigonometric function. Solution will be obtained for each order n separately, where the integra n denotes the circumferential order and the mode label. So the displacement field can be written as

$$\bar{\boldsymbol{u}} = \bar{\boldsymbol{u}}_r e^{-i(\omega t - kz - n\theta)} \tag{9}$$

In the SBFEM of the cylindrical waveguide, a scaling coordinate $\bar{z}$ parallels to the z direction and measures the length from the scaling center at infinity. The geometry of a hollow cylinder is described by one-dimensional finite element with local coordinate η in the radical direction only. The following steps are derived for one isoparametric element to obtain the stiffness matrix. The geometry of a finite element is represented by interpolating its nodal coordinates r_i using the local coordinates η:

$$r(\eta) = N_i(\eta)r_i \tag{10}$$

where $N_i(\eta)$ is the mapping function. To transform the differential operator of the Cartesian coordinate to the local coordinate, the relationship is required

$$\begin{matrix} x = \cos(\theta)r(\eta) \\ y = \sin(\theta)r(\eta) \end{matrix} \quad \begin{bmatrix} \partial_\theta \\ \partial_\eta \end{bmatrix} = \boldsymbol{J}\begin{bmatrix} \partial_x \\ \partial_y \end{bmatrix} \tag{11}$$

with the Jacobian matrix

$$\boldsymbol{J} = \begin{bmatrix} -\sin(\theta)r(\eta) & \cos(\theta)r(\eta)_{,\eta} \\ \cos(\theta)r(\eta) & \sin(\theta)r(\eta)_{,\eta} \end{bmatrix} \tag{12}$$