Composite Structures 132 (2015) 597–609

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Mechanical and thermal stability of eccentrically stiffened functionally graded conical shell panels resting on elastic foundations and in thermal environment

Nguyen Dinh Duc [a,*], Pham Hong Cong [a], Vu Minh Anh [a], Vu Dinh Quang [a], Phuong Tran [b], Ngo Duc Tuan [b], Nguyen Hoa Thinh [c]

[a] Vietnam National University, Hanoi, 144 Xuan Thuy, Cau Giay, Hanoi, Viet Nam
[b] The University of Melbourne, Parkville 3010, VIC, Australia
[c] Vietnam Mechanics Society, 264 Doi Can, Ba Dinh, Hanoi, Viet Nam

ARTICLE INFO

Article history:
Available online 6 June 2015

Keywords:
Mechanical and thermal stability
Eccentrically stiffened FGM conical shell panel
Temperature-dependent properties
Elastic foundations
Thermal environment

ABSTRACT

Conical shell panels made of functionally graded materials (FGMs) are rather commonly used by structural engineers. However, due to their complex geometric shape, there are only a few studies on conical shell panels made from FGMs. This paper investigates the linear stability analysis of eccentrically stiffened FGM conical shell panels reinforced by mechanical and thermal loads on elastic foundations. The FGM conical shell is in thermal environment and both the panel and the stiffeners are deformed under temperature. The material properties of both the panels and stiffeners are assumed to be temperature-dependent. Classical shell theory and Lekhnitsky's smeared stiffeners technique are used to set the balance equations and linear stability. Shells are reinforced by stringers and rings. The effects of stiffeners, material, and mechanical and temperature loads on stability of the eccentrically stiffened FGM conical shell panels are analysed and discussed.

1. Introduction

Functionally graded materials (FGM), which are microscopic composites made from a mixture of metal and ceramic constituents, were first introduced in 1984 by a group of Japanese materials scientists [1]. Functionally graded materials involving conical shells are widely used in exhaust nozzles of solid rocket engines, space vehicles, aircrafts, nuclear power plants, and many other engineering applications.

The static stability of conical shells has been studied by many researchers in recent years. However, most of them have focused on buckling behaviours and determining the critical loads for shells without elastic foundations and stiffeners. The thermal and mechanical instability of functionally graded truncated conical shells have been investigated by Naj et al. [2]. Sofiyev [3] has presented thermoelastic stability of functionally graded truncated conical shells. In [4–6], Sofiyev et al. have considered the buckling of thin truncated conical shells made of FGMs subjected to hydrostatic pressure, uniform external pressure, and uniform axial compressive load. In [7], Sofiyev and Kuruoglu investigated the buckling problem of FGM truncated conical shells subjected to external pressures under mixed boundary conditions. Sofiyev [8] investigated the non-linear buckling of imperfect FGM truncated conical shells with simply supported boundary conditions and subjected to an axial compressive load. In this work [8], the numerical illustrations demonstrated the non-linear buckling response of FGM truncated conical shells with different shell parameters, initial imperfections and compositional profiles. An analytical approach to investigate the linear buckling of truncated conical panels made of FGM and subjected to axial compression, external pressure, and a combination of these loads has been investigated by Bich et al. [9]. Therefore, studies on the effects of elastic foundations on behaviour and loading capacity of the shells are very important. The non-linear buckling of the truncated conical shells made from FGMs surrounded by an elastic medium and Winkler-Pasternak type elastic foundations was presented in [10,11]. Other studies about vibration of FGM conical shell panels, such as investigation offree vibration of FGM conical shell panels, have been investigated by Akbari et al. in [12] and Zhao et al. in [13].

Composite plates and shells are usually reinforced by stiffening components to provide the benefits of added load-carrying static and dynamic capability, with a relatively small additional weight. There have been some publications of Bich et al. [14], Dung et al.

* Corresponding author.
E-mail address: ducnd@vnu.edu.vn (N.D. Duc).

http://dx.doi.org/10.1016/j.compstruct.2015.05.072
0263-8223/

Nomenclature

h	thickness of conical shell panels
L	length of conical shell panels
α	semi-vertex angle
β	open angle
a	small base radius
h_s, b_s	thickness and width of stringer (x-direction), subscript s stand for the stringer stiffeners constituent.
h_r, b_r	thickness and width of ring (θ-direction), subscript r stand for the ring stiffeners constituent.
d_s, d_r	distance between two stringers and two rings
z_s, z_r	eccentricities of stiffeners with respect to the middle surface of shell
K_1	Winkler foundation stiffeness (N/m^3)
K_2	shear subgrade modulus of the Pasternak foundation model (N/m)
$\varepsilon_x^0, \varepsilon_y^0$	normal strains
γ_{xy}^0	shear strain at the middle surface of the shell
$k_x, k_\theta, k_{x\theta}$	change of curvatures and twist, respectively
$E_s, \upsilon_s, \alpha_s$	Young's modulus, Poisson's ratio and thermal expansion coefficient of the stiffener in the x-direction, respectively
$E_r, \upsilon_r, \alpha_r$	Young's modulus, Poisson's ratio and thermal expansion coefficient of the stiffener in the θ-direction, respectively
$n_s,\ n_r$	number of stringer and ring, respectively
A_s, A_r	cross-sectional area of stiffeners
I_s, I_r	second moments of inertia of the stiffener cross-sections related to the shell middle surface
A_s^T, A_r^T	although the stiffeners are deformed by temperature, we have assumed that the stiffeners keep their rectangular shape of the cross-section.

[15,16] and Duc et al. [17–20] on the buckling of composite shells reinforced by stiffeners. A semi-analytical approach to investigate the non-linear dynamic effect of imperfect eccentrically stiffened FGM shallow shells taking into account damping when subjected to mechanical loads was conducted in [14]. Dung et al. [15] have investigated the instability of eccentrically stiffened functionally graded truncated conical shells under mechanical loads where the shells are reinforced by stringers and rings. Also, Dung et al. [16] have investigated the stability of functionally graded truncated conical shells reinforced by functionally graded stiffeners, surrounded by an elastic medium and under mechanical loads. The non-linear static buckling and post-buckling for imperfect eccentrically stiffened functionally graded thin cylindrical shells on elastic foundations has been investigated by Duc and Thang [17].

From the above review, to the best of our knowledge, there are only a few publications about buckling of FGM shells with stiffeners in thermal environments. Under temperature, both the FGM shells and the stiffeners are deformed, therefore, determination of the thermal mechanism of FGM shells and stiffeners becomes more difficult. Recently, in [18,19], Duc et al. analysed the non-linear post-buckling of imperfect eccentrically stiffened FGM plates and FGM double-curved thin shallow shells on elastic foundations using a simple power-law distribution in thermal environments. In [20], Duc et al. also studied the thermal stability of eccentrically stiffened functionally graded truncated conical shells in thermal environments.

This paper studied the stability of eccentrically stiffened FGM conical shell panels (as segment of the truncated conical shells) on elastic foundations under mechanical and thermal loads, with both the FGM shell and stiffeners having temperature-dependent properties. The paper also analysed and discussed the effects of material, temperature, elastic foundations, and eccentric stiffeners on critical buckling loads of the FGM conical shell panels, and the material properties were assumed to be temperature-dependent.

2. Eccentrically stiffened FGM conical shell panels on elastic foundations

We consider FGM conical shell panels having thickness h, length L, and semi-vertex angle α, limited by two circles with radii of $a$and b (bottom). Dividing the conical shell by two planes containing the axis with opening angle $\beta(\beta \neq 2\pi)$, we get pieces of the shell called conical open panel. On the contrary, when the opening angle $\beta = 2\pi$, we get a problem with the truncated conical shell, which was solved in [20].

Conical shell panels made from a mixture of ceramic and metal was defined in a coordinate system (x, θ, z), with the origin placed in the centre of the panel, the axis $x$along the generatrix direction from the top of the cone shell, the axis θ along the parallel of latitude direction, and the axis z perpendicular to the medium and outward, h_s and b_s are the thickness and width of stringer stiffeners (x-direction); h_r and b_r are the thickness and width of ring stiffeners (θ-direction). Also, $d_s = d_s(x)$ and d_r are the distance between two stringers and two rings,respectively. z_s, z_r represent the eccentricities of stiffeners with respect to the middle surface of shell (Fig. 1).

The effective properties of the FGM conical shell panels (the elastic modulus, Poisson's ratio, and thermal expansion coefficient α) can be written as follows[21]:

$$\begin{Bmatrix} E \\ \alpha \end{Bmatrix} = \begin{Bmatrix} E_m \\ \alpha_m \end{Bmatrix} + \begin{Bmatrix} E_{cm} \\ \alpha_{cm} \end{Bmatrix} \left(\frac{2z+h}{2h} \right)^N, \tag{1}$$

where $E_{cm} = E_c - E_m, \alpha_{cm} = \alpha_c - \alpha_m$; the volume fraction index N is a non-negative number that defines the material distribution and can be chosen to optimise the structural response; and subscripts m and c stand for the metal and ceramic constituents, respectively. Poisson's ratio is assumed to be constant, $v = const$. From Eq. (1) we have: $E = E_m$ at $z = -h/2$ (metal-rich), and $E = E_c$ at $z = h/2$ (ceramic-rich).

The material property (Pr) of both the FGM conical shell panels and stiffeners, such as the elastic modulus E, Poisson's ratio v, and thermal expansion coefficient α, can be expressed as a non-linear function of temperature [22], as follows:

$$\Pr = P_0\left(P_{-1}T^{-1} + 1 + P_1T^{-1} + P_2T^2 + P_3T^3\right), \tag{2}$$

in which $T = T(z) = T_0 + \Delta T(z)$ and $T_0 = 300$ K (room temperature); are coefficients characterizing the constituent materials; ΔT is the temperature rise from stress free initial state, and more generally, $\Delta T = \Delta T(z)$. In short, we will use T-D (temperature-dependent) for cases where the material properties depend on temperature. Otherwise, we will use T-ID for temperature-independent cases. The material properties for the latter one have been determined by Eq. (2) at room temperature, i.e $T_0 = 300$ K.

The FGM conical shell panels are on elastic foundations (Fig. 1). The Pasternak model is used to describe the reaction of the elastic foundations on the conical shell panels. If the effects of damping