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Temperature-dependent nonlinear behaviour of thin rectangular plates exposed to through-depth thermal gradients

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ABSTRACT

A theoretical model is developed for geometrically and materially nonlinear analysis of thin rectangular plates subjected to transverse mechanical loads and exposed to non-uniform thermal gradients over their depth. The geometrical nonlinearity is based on the von Kármán type of large deformation theory. The material nonlinearity arises from degradable material behaviour at elevated temperatures. The temperature distribution is obtained numerically for two common types of fire exposure conditions that could occur in a fire compartment including: an exponential "short hot" fire leading to a high temperature over a relatively short duration; and an exponential "long cool" fire of lower temperature over a longer duration. Two types of support conditions are considered for the plate based on assuming that in-plane displacements are either restrained or unrestrained against lateral translation. Several numerical examples including two examples for functionally graded plates are presented to assess the accuracy and performance of the proposed method. The evolution of the true shape of the compressive zone supporting tensile membrane action in laterally unrestrained plates under large displacements is graphically illustrated for the two non-uniform thermal gradients. It is shown that the effect of the short hot fire on the plate behaviour is more pronounced.

1. Introduction

In most engineering structures, there are circumstances where structural components are exposed to non-uniform thermal gradients while externally loaded, e.g., when oil platforms or large compartments in buildings are subjected to fire conditions, or when aerospace vehicles are subjected to aerodynamic heating upon reentry into earth atmosphere. Such loading conditions typically induce two effects on the structure, deformation or geometry change (due to thermal expansion) and reduction of strength and stiffness of the structure (due to material degradation). As a consequence, there are two concurrent actions associated with thermally-induced displacements and load-induced displacements. At an early stage of thermal exposure, the structural behaviour is dominated by the latter, however, close to structure failure when material properties have significantly degraded, it is dominated by the former. Such a mechanism was proposed by Usmani et al. [1] to explain how and why the Twin Towers of the World Trade Center in New York collapsed as a result of the terrorist attack. However, full exploitation of the new understanding developed from that research requires further more detailed investigations, so it could lead to designing safer structures.

Considering the complexity of such problems, recourse is made to numerical techniques. Barut et al. [2] presented a nonlinear finite element (FE) formulation for moderately thick flat and curved laminated panels subjected to non-uniform thermal loading over the panel surface and across its thickness. The material properties were assumed independent of the temperature change. The principle of virtual work, along with the co-rotational form of the total Lagrangian formulation were used to derive the governing equations. Jin et al. [3] performed an FE investigation of thermal post-buckling behaviour for patched cylindrical composite panels under uniform and non-uniform temperature distributions through the thickness direction. Their analysis was based on the Hellinger–Reissner principle. Na and Kim [4] studied the nonlinear bending of laterally restrained plates made of functionally graded materials (FGMs) subjected to uniform pressure and thermal loads using a 3D FE method. The thermal loads were assumed to be uniform, linear and quarter sine wave temperature distributions across the plate thickness. Sabik and Kreja [5] numerically

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investigated the load capacity of thermally loaded multi-layered plates and shells under uniform thermal loading. Their formulation was based on the first order shear deformation theory, but the thermal degradation of material properties was not taken into account in their model. Another FE analysis was recently performed by Jeyaraj [6] to study the critical buckling temperature and free vibration modes of isotropic plates under arbitrary varying temperature fields. More recently, Salminen and Heinisuo [7] proposed a design method for predicting shear resistance of thin steel plates at non-uniform elevated temperatures. Nonlinear behaviour of steel at elevated temperatures was considered in their model according to Eurocode [8].

Along with advances in numerical methods, a significant improvement is also observed in the capability of analytical and semi-analytical approaches in capturing large deformations of plates and shells. Shukla and Nath [9] used an analytical technique to investigate the steady-state response of moderately thick laminated composite rectangular plates under various boundary conditions (BCs), including simply supported immovable edges, clamped immovable edges, free edges, and their combinations undergoing moderately large deformations. The method of solution was based on a Chebyshev series solution technique. Woo and Meguid [10] studied the nonlinear analysis of laterally unrestrained thin rectangular plates and shallow shells with FGM properties subjected to a transverse mechanical load and a temperature field through the thickness direction. The governing equations were established based on the von Kármán theory for large transverse deflections and were solved using series solutions. Shen [11] presented an analytical method for thermal post-buckling analysis of a simply supported shear deformable functionally graded (FG) plate under both in-plane non-uniform parabolic temperature distribution and heat conduction based on a higher-order shear deformation plate theory. Li et al. [12] analytically studied the geometrically nonlinear deformation of clamped imperfect circular FG plates subjected to both mechanical load and non-uniform temperature increase over the depth of the plate. Recently, Sepahi et al. [13] investigated the effects of thermal and combined thermo-mechanical loadings on axisymmetric large deflection of a simply supported annular FG plate resting on an elastic foundation. The thermal loading was assumed to be non-uniform over the depth of the plate.

In a general sense, numerical methods have been widely adopted by the scientific community to cope with both geometrical and material nonlinearities in thermo-mechanical modelling of plate and shell structures. There is no doubt that such methods provide greater flexibility in analysing structures when compared to closed form analytical methods. It is nevertheless possible to benefit from adequately accurate analytical or semi-analytical approaches that take into account all the key features and complexities of the problem, when computational effort (mostly in terms of analyst effort) is a concern, or when an alternative approach is required to validate and corroborate numerical results. These approaches not only provide rapid alternatives and benchmark solutions for assessing the accuracy and validity of numerical results, they could potentially also provide advanced basis functions for hybrid-type computational approaches. For example, in the context of structures in fire, benchmark solutions can identify interesting interactions between heated structural members and the cooler surrounding structure which sometimes result in sudden changes that are difficult to model numerically.

The application of this concept to various composite structures seems straightforward, however, little work exists in this area. We aim to apply this strategy to plate structures under more realistic thermal loading conditions by simulating extreme thermo-mechanical loads such as fire, through an ongoing research project at the University of Edinburgh in the UK [14,15]. In most composite plate analyses, particularly for analysis of FG plates, spatially varying material properties are not employed and the analysis has been limited to temperature sensitive material properties.

Hence in this paper, the nonlinear structural behaviour of temperature-dependent rectangular plates subjected to transverse mechanical loads and non-uniform thermal loads is modelled. The thermal loading includes two different fire conditions, one representing a “short hot” exponential fire of high temperature over a short post-flashover duration, while the other represents a “long cool” exponential fire with lower maximum temperature over a longer post-flashover period. Both geometrical and material nonlinearities are included in the model. The geometrical nonlinearity is based on the von Kármán type of large deformation theory, while the material nonlinearity arises from considering the reduction of the plate’s mechanical and thermal properties at elevated temperatures. BCs are such that rotations parallel to the plate boundaries are assumed to be free while lateral translations across the boundaries may be free or restrained. The out-of-plane (or transverse) displacement at the plate boundaries is always restrained. The analysis is carried out assuming quasi-static conditions ignoring any dynamic effect. The difference between the two limiting cases (rigid restraint or zero restraint) under thermal loadings on the plate response is highlighted in the results section. In the case of laterally unrestrained plates undergoing large displacements, the evolution of the shape of the compressive zone supporting tensile membrane action (TMA) is graphically illustrated for the two non-uniform thermal gradients while also considering nonlinear temperature-dependent material behaviour. The accuracy of the present method is investigated through several numerical examples, including two examples for FG plates.

2. Fundamental relations

A rectangular plate of length a, width b, and uniform thickness h subjected to a transverse mechanical load and a non-uniform thermal gradient caused by a heat source across the thickness of the

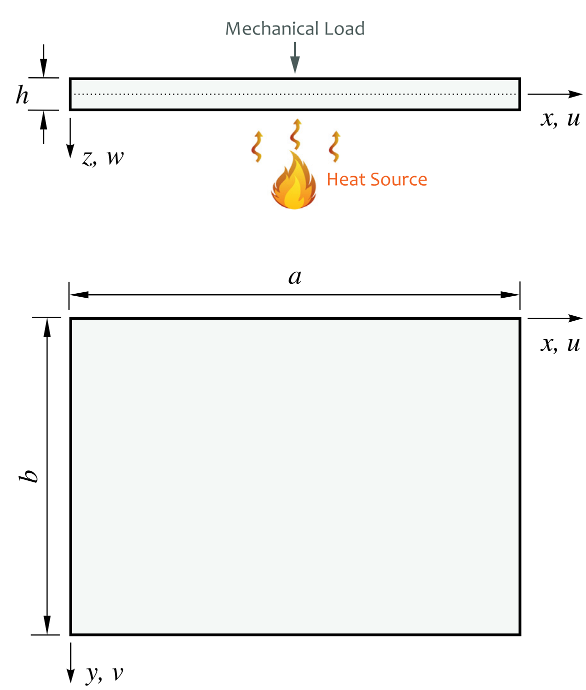

Fig. 1. Configuration of a rectangular plate under a transverse mechanical load and a non-uniform thermal gradient caused by a heat source such as fire. For plate structures under fire conditions, interest lies mainly in the variation of temperature through the depth of the plate.