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Prediction of load–deflection behavior of multi-span FRP and steel reinforced concrete beams

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ABSTRACT

This paper presents a numerical procedure to determine the deflection of concrete members reinforced with fiber reinforced polymer (FRP) or steel bars. This procedure is implemented into the stiffness matrix to allow for general use in the structural analysis. It considers effective flexibilities of members in the cracked state using either the curvature distribution along the member or available effective stiffness models under any loading or support condition. In general, structural concrete members can be considered to have three cracked regions (two at the ends and one at midspan) and two uncracked regions along their length. In this numerical procedure, the contributions of these regions to the member stiffness matrix are computed using a numerical integration technique. Using this procedure, a software program is developed which allows for the load–deflection behavior of a member reinforced with either FRP or steel bars and subjected to any loading or support condition to be rapidly determined. This calculation procedure is evaluated using available experimental data on the load–deflection behavior of simple and two-span beams reinforced with FRP and steel bars. Through comparison of the results, it is observed that the load–deflection behaviors calculated using the proposed approach utilizing the member moment–curvature response are consistent with the experimental data. This approach can provide a useful tool for the general calculation of deflection regardless of reinforcement type and can be used throughout the entire range of member behavior up to flexural failure.

1. Introduction

The use of fiber reinforced polymer (FRP) bars in structural concrete has rapidly increased in the last two decades due to their superior durability, excellent corrosion resistance, non-magnetic properties, and high strength-to-weight ratio compared to conventional steel bars. On the other hand, FRP bars have a lower modulus of elasticity compared to steel. Because of this fact, the same amount of reinforcement exhibits larger deflections and crack widths in FRP reinforced concrete members than in steel reinforced concrete members. Hence, the design of such members is typically governed by the serviceability limit state, and accurate determination of deflection becomes very important.

Recently, there has been extensive research to investigate the flexural behavior of FRP reinforced concrete members [1–17]. Many design approaches to calculate the deflection of FRP reinforced concrete members have been proposed in these studies. For the case of service level deflections, some authors have presented additional coefficients taking into account the specific properties of FRP bars. These coefficients are used to modify Branson's equation, which is semi-empirical and commonly used for steel reinforced concrete members in design codes [18], to compute the effective moment of inertia of members reinforced by FRP bars. Others have presented a modified equivalent moment of inertia derived from assumed moment–curvature relationships of the FRP reinforced concrete [19–27]. These models, however, are not completely generalized to work with all loading and support conditions.

The purpose of this research is to develop a numerical procedure that can be implemented into the stiffness matrix in the structural analysis so that the deflection of concrete members reinforced with FRP or steel bars can be determined under any loading or support condition. In this procedure, member flexibility is determined using either the moment–curvature relationship of the reinforced concrete section obtained from a cracked-section analysis or available semi-empirical effective stiffness models. A software program was developed using this procedure which allows the load–deflection behavior of a member reinforced with

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either FRP or steel bars to be rapidly calculated for any loading or support condition.

2. Effective flexibility models for cracked members

To determine the deflection of concrete members reinforced with FRP bars, the effective flexibility of the cracked member is required. In the literature, the effective flexibility of cracked members is calculated using different semi-empirical equations as follows:

ACI 440.1R [28,29]:

$$\frac{1}{E_cI_{eff}}=\frac{1}{E_cI_{cr}}\left[1-\frac{\omega}{1+\omega}\right]\leqslant\frac{1}{E_cI_g}\quad\text{for } M\geqslant M_{cr}\tag{1}$$

where:

$$\omega=\left(\frac{M_{cr}}{M}\right)^3\left(\frac{\beta_dI_g}{I_{cr}}-1\right)\tag{1a}$$

$$\beta_d=\alpha_b\left(\frac{E_f}{E_s}+1\right);\quad \alpha_b=0.5\quad\text{for ACI 440.1R}-03\tag{1b}$$

$$\beta_d=0.2\frac{\rho_f}{\rho_{fb}}\quad\text{for ACI 440.1R}-06\tag{1c}$$

where E_c, E_f, and E_s are the modulus of elasticity of concrete, FRP, and steel reinforcement; M is the applied bending moment; M_{cr} is the flexural cracking moment of the section; I_g and I_{cr} are the moments of inertia of the gross and cracked transformed section; and ρ_f and ρ_{fb} are the FRP tensile reinforcement ratio and FRP reinforcement ratio producing balanced strain conditions, respectively.

ACI 440-H [17]:

$$\frac{1}{E_cI_{eff}}=\frac{1}{E_cI_{cr}}\left[1-\eta\gamma\left(\frac{M_{cr}}{M}\right)^2\right]\leqslant\frac{1}{E_cI_g}\quad\text{for } M\geqslant M_{cr}\tag{2}$$

where:

$$\eta=1-\frac{I_{cr}}{I_g},\quad \gamma=1.72-0.72\left(\frac{M_{cr}}{M}\right)\tag{2a}$$

Bischoff [21], ***ISIS*** [30] ***and CEB*** [31]

$$\frac{1}{E_cI_{eff}}=\frac{1}{E_cI_{cr}}\left[1-\beta_1\beta_2\left(\frac{M_{cr}}{M}\right)^2\left(1-\frac{I_{cr}}{I_g}\right)\right]\leqslant\frac{1}{E_cI_g}\quad\text{for } M\geqslant M_{cr}\tag{3}$$

where:

$\beta_1\beta_2=1.0$ in Bischoff, $\beta_1\beta_2=0.5$ in ISIS and $\beta_1\beta_2=0.8$ in CEB models.

Benmokrane et al. [1]

$$\frac{1}{E_cI_{eff}}=\frac{1}{E_cI_{cr}}\left[1-\frac{(\alpha-1)I_{cr}+\psi}{\alpha I_{cr}+\psi}\right]\leqslant\frac{1}{E_cI_g}\quad\text{for } M\geqslant M_{cr}\tag{4}$$

where:

$$\psi=\left(\frac{M_{cr}}{M}\right)^3\left(\frac{I_g}{\beta}-\alpha I_{cr}\right)\quad \beta=7;\quad \alpha=0.84\tag{4a}$$

Yost et al. [8]:

Same as Eq. (1b) except $\alpha_b=0.064\left(\frac{\rho_f}{\rho_{fb}}\right)+0.13$.

Toutanji and Saafi [4]:

$$\frac{1}{E_cI_{eff}}=\frac{1}{E_cI_{cr}}\left[1-\frac{\left(\frac{M_{cr}}{M}\right)^m\left(\frac{I_g}{I_{cr}}-1\right)}{1+\left(\frac{M_{cr}}{M}\right)^m\left(\frac{I_g}{I_{cr}}-1\right)}\right]\leqslant\frac{1}{E_cI_g}\quad\text{for } M\geqslant M_{cr}\tag{5}$$

where:

$$m=6-10\left(\frac{E_f}{E_s}\right)\rho_f\quad\text{if }\left(\frac{E_f}{E_s}\right)\rho_f<0.3$$

$$m=3\quad\text{if }\left(\frac{E_f}{E_s}\right)\rho_f\geqslant 0.3$$

For all models, the effective flexibility of the member is:

$$\frac{1}{E_cI_{eff}}=\frac{1}{E_cI_g}\quad\text{for } M<M_{cr}\tag{6}$$

The models listed above for calculating the flexibility of a FRP reinforced concrete member are derived from the semi-empirical Branson's equation [32]. In these models, the stress–strain relationships of the concrete and reinforcement are not considered. Instead, empirically derived correction factors are used to improve the deflection prediction of the models.

Other models available in the literature for calculating deflections are based on interpolation between a fully cracked and uncracked state of the member using the curvature distribution. In these models, closed form equations for deflection calculations are developed for only specific member boundary conditions and loading types [19–25].

The present study proposes the use of the complete moment–curvature relationship obtained from a sectional analysis and taking into account the stress–strain relationships of the concrete and reinforcement. A general purpose software program was also developed based on the stiffness matrix formulation of the member in either the cracked or uncracked state.

3. Moment–curvature analysis

Moment–curvature relationships are developed for rectangular concrete sections reinforced with either steel or FRP reinforcement. For this analysis, it is considered that the reinforcement may be in either tension or compression regions. The following material constitutive laws are considered:

3.1. Concrete stress–strain models

Any model for concrete in compression can be used in the analysis procedure. For example, if the CEB-FIB model [33] is used, the following equations are considered:

$$f_c=f'_c\left[\frac{2\varepsilon_c}{\varepsilon_{co}}-\left(\frac{\varepsilon_c}{\varepsilon_{co}}\right)^2\right]\quad \varepsilon_c\leqslant\varepsilon_{co}\tag{7a}$$

$$f_c=f'_c\quad \varepsilon_{co}\leqslant\varepsilon_c\leqslant\varepsilon_{cu}\tag{7b}$$

where f_c and ε_c are the compressive stress and strain in concrete; f'_c is the cylinder compressive strength of concrete; and ε_{co} and ε_{cu} are the strain in concrete at maximum stress and the ultimate strain of concrete as shown in Fig. 1(a) and (b). If the Hognestad model is assumed, only Eq. (7a) is used with the concrete strain $\varepsilon_c\leqslant\varepsilon_{cu}$.

Any model of the tensile stress–strain relationship of concrete can also be used. If a bilinear stress–strain relationship is used, the following equations are considered:

$$f_t=E_c\varepsilon_t\quad \varepsilon_t\leqslant\varepsilon_{cr}\tag{8a}$$

$$f_t=f_r-\frac{f_r}{\varepsilon_{ctu}-\varepsilon_{cr}}(\varepsilon_t-\varepsilon_{cr}),\quad \varepsilon_{ctu}\geqslant\varepsilon_t\geqslant\varepsilon_{cr}\tag{8b}$$

$$\varepsilon_{ctu}=\alpha_{ts}\varepsilon_{cr}\tag{8c}$$

where f_t and ε_t are the tensile stress and strain of the concrete; E_c is the tensile modulus of concrete, assumed to be same as the modulus of elasticity in compression; f_r and ε_{cr} are the modulus of rupture of the concrete and the corresponding cracking strain; and ε_{ctu} is the ultimate tensile strain of the concrete, assumed to be α_{ts} times of the cracking strain (ε_{cr}) as shown in Fig. 1(c). The parameter α_{ts} controls tension stiffening which affects the moment–curvature relationships particularly at initial cracking stages of