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A general formulation of quality factor for composite micro/nano beams in the air environment based on the nonlocal elasticity theory

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ABSTRACT

The quality factor is known as a critical and important feature to achieve efficient resonant response of micro/nano beams. In fact, the quality factor plays a significant role in the sensitivity and resolution of resonant vibration behavior of small-size structures. The presented paper investigates the quality factor of composite micro/nano beams employing the nonlocal Euler–Bernoulli beam theory. Composite beams having arbitrary laminated layers and discontinuities are taken into consideration. Regarding to size effects, a general formulation is derived to calculate the quality factor of the nonlocal beams attributed to airflow damping and support losses. To reduce complexity of the problem, a nondimensional parameter is introduced to calculate the quality factor. To provide guidelines for determining the quality factor of the nonlocal composite beams, general results are presented for several resonant modes of vibration. The obtained results indicate that the quality factor is decreased by increasing the nonlocal size effects. However, the size effects play more prominent role at the higher resonant modes of vibration. Furthermore, it is shown that the quality factor is affected by the boundary conditions and dimensional characteristics of the micro/nano composite beams. Accordingly, the quality factor is increased by decreasing the slenderness ratio of the beam.

1. Introduction

In recent decades, new small devices known as micro/nano structures have been emerged to engineering fields due to advanced techniques in manufacturing and fabrication [1–4]. These innovative small structures exhibit essential and imperative role in the modern technologies because their mechanical features are extremely improved by decreasing their dimensions. Therefore, these small structures are widely implemented in several micro/nano systems such as atomic force microscope (AFM), micro/nano switches, micro/nano resonators and bio-sensors [5–8]. Currently, micro/nano beam-like structures are identified as basic constituents of these small systems because of several problems in manufacturing joints in micro-to-nano scale [9]. Furthermore, micro/nano beams exhibit several novel features like simple manufacturing and wide-bandwidth frequency operation [10].

In micro-to-submicron scales, beam motion is originated from their displacement and deflection in the response of external forces. However, micro/nano beams can operate in static or dynamic mode, but in the most applications, they are operated in resonant modes of vibration [11]. Thus, resonant vibration characteristics such as frequency response and sensitivity of such small-size beams play a crucial role in enhancing their performance and efficiency. Moreover, the vibrating behavior of the resonant micro/nano beams is extremely relevant to their quality factors. The quality factor is a key parameter to improve response and sensitivity of the beam in infinitesimal scales [12]. The higher quality factor causes more stable frequency response, better sensitivity and resolution, and the lower sensing noise in micron and sub-micron scales [12,13]. Therefore, it is essential to achieve higher values of the quality factor for resonant micro/nano beams.

Generally, the quality factor is defined as a measurement criterion of energy dissipation to its stored energy in a resonant structure. The quality factor can be related to several energy dissipation mechanisms e.g. hydrodynamic damping exerted by an ambient environment, support losses, thermoelastic damping, surface and volume losses [14–17]. However, the quality factor of micro/nano beams operated in air environments are mainly affected by the viscous airflow damping comparing to other energy dissipation mechanisms [17–19]. The air damping is attributed to energy dissipation of the resonant small beams in interaction with ambient air environments. Thus, the air damping and corresponding quality factor is related to resonant frequency, dimensional and material properties of the micro/nano beams. Accordingly, the higher

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quality factor and consequently less energy dissipation can be achieved by appropriate material selection of the micro/nano beam. However, it is restricted by typical affordable materials in micro/nano fabrication technology [20].

In addition, dimensional and geometrical properties of the micro/nano beams have great influence on their resonant vibrating frequencies and quality factors in air surroundings. Dimensional effects on the quality factor of silicon microcantilever beams in air environments are presented in [18]. In this work, the effect of the beam dimensions especially length and thickness on the quality factor at the fundamental resonance vibration is only studied. Lee et al. [19] presented a numerical approach to investigate dimensional effects on the quality factor of uniform micro beams in free air space. They considered the classical continuum model of the microcantilever beams and calculated their quality factor attributed with air viscous damping numerically and experimentally. Damircheli and Korayem [21] investigated effects of geometrical properties of an atomic force microscope microcantilever on its resonant response in air environments. However, they developed vibration equations of the beam based on classic continuum mechanics and studied first four flexural modes of vibration.

As it can be seen from the literature, several researches have been studied the quality factor and resonant vibrating behavior of micro/nano beams using classical elasticity theory. On the other hand, experiments and molecular simulations in micro/nano scale have demonstrated effects of the size-dependent mechanical behavior on micro/nano systems [22–24]. Zhang et al., [25] employed molecular dynamics simulation to investigate damping effects on the quality factor of a nano-resonator and showed such effects are excluded in the analytical model based on classical continuum theory. In fact, in micron and sub-micron scales, material microstructures are noticeably important and the classic continuum theory does not enable to interpret it. On the other hand, in micro and nano scales, molecular simulations are suffered from excessive computations and experiments are extremely difficult as well. Therefore, the size-dependent elasticity theories have emerged into micro/nano researches and they obtained a great deal of scientific interests. Civalek and Akgöz [26] investigated size-dependent free vibration of non-uniform composite micro beams based on modified couple stress theory. Jiang and Yan [27] employed the surface elasticity theory to study static behavior of nanowires. They presented explicit solutions to analyze surface effects on deflection of the beam. Zhang et al. [28] modeled and analyzed size effects of uniform micro beams implementing modified couple stress theory. They showed the size effects on buckling behavior of the beam. Akgöz and Civalek [29] studied longitudinal free vibration of a micro-scaled bar based on the strain gradient elasticity theory. They showed that the difference between natural frequencies predicted by current and classical models becomes more prominent for higher modes of vibration. Also, they investigated buckling behavior of size-dependent microbeams made of functionally graded materials. In this paper, different boundary conditions on the basis of Bernoulli–Euler beam and modified strain gradient theory are considered [30].

In addition, Eringen [31] proposed nonlocal continuum mechanics formulation which has extensive potential applications to investigate the size effect on small-scale systems. Peddison et al. [32] revealed the potential of the nonlocal elasticity method for size-dependent modeling of micro/nano systems. Along this line, the nonlocal theory is presented in [33] to analyze buckling and vibration of nonlocal simply supported beams. Civalek and Demir [34] presented bending formulation of microtubules based on nonlocal Euler–Bernoulli beam theory. Then, they implemented differential quadrature method to solve the problem numerically. Also, they studied free vibration analysis of uniform cantilever microtubules based on nonlocal continuum model [35]. They presented some numerical results to show the effect of small-size effects on bending and vibration of micro beams. Nazemizadeh and Bakhtiari-Nejad [36] investigated size-dependent free vibration of piezoelectric actuated beams based on the nonlocal elasticity theory. They developed an analytical solution and studied size effects on natural frequencies of the system.

Although the number of researches have studied the quality factor of uniform resonant micro/nano beams based on classical continuum models in air environment, but to the best of our knowledge there is not any research work available in the literature that apply the concept of size effects to resonant characteristics of micro/nano beams. Therefore, there is a need to analyze size effects on the quality factor of the resonant micro/nano beams employing size-dependent elasticity theories. This paper presents a general approach to determine the quality factor and resonant characteristics of non-uniform composite micro/nano beams in air environments based on nonlocal continuum models. The governing equations of the nonlocal composite beam with arbitrary laminated layers along its thickness and discontinuities along its length are derived. To formulate the quality factor, the airflow damping and support losses are taken into account. By employing various nonlocal boundary conditions, the quality factor of the composite beam is formulated. Then, a number of simulations are carried out to evaluate the quality factor of the beam at various resonances in air environments. Also, nonlocal and dimensional effects on the quality factor of composite beams are studied and simulation results are compared and discussed.

2. Nonlocal composite nano/micro beam

In this section, governing vibrating equation of a general composite micro/nano beam is derived based on the nonlocal continuum theory. The pioneering works on the nonlocal continuum mechanics were reported by Eringen [37,38]. Then the nonlocal elasticity theory has accepted and applied by several researchers in the field of micro- and nanotechnology [34–36]. Moreover, this theory has been compared with molecular dynamic simulations and a good agreement has been observed [39].

Despite of the classical continuum mechanics, the nonlocal elasticity theory considers that the stress components at a given point are not only a function of the strain components of the same point but also to all other points of a medium. Therefore, for homogenous and isotropic elastic solids, the nonlocal elasticity theory is formulated by the following equations [37]:

$$\sigma^{nl}_{kl,l} + \rho\left(f_k - \frac{\partial^2 u_k}{\partial t^2}\right) = 0 \tag{1}$$

$$\sigma^{nl}_{kl}(\vec{r}) = \int_V \alpha(|\vec{r} - \vec{r}'|, \chi)\sigma^{l}_{kl}(\vec{r}')dV(\vec{r}') \tag{2}$$

where σ^{nl}_{kl} is the nonlocal stress tensor, ρ is the mass density of the body, $\vec{r}$ denotes a reference point, f_k is the body force density, $\vec{u}$ is the displacement vector at the reference point $\vec{r}$ in the body, $\sigma^{l}_{kl}(\vec{r}')$ indicates the classical stress tensor at any point $\vec{r}'$ in the body, V denotes the volume occupied by the body, $|\vec{r} - \vec{r}'|$ is the distance in Euclidean form and χ is defined as a material constant. The nonlocal kernel $\alpha|x - x'|$ is stated as the influence of the strain at the point $\vec{r}'$ on the stress at the point $\vec{r}$ in the elastic body. Thus, by defining an appropriate nonlocal kernel, the nonlocal constitutive given by Eq. (2) is reduced to the differential equation [37]:

$$\Re\sigma^{nl}_{kl} = \sigma^{l}_{kl} \tag{3}$$

where the differential operator is assumed as $\Re = 1 - \tau^2\nabla^2$, ∇^2 is the Laplacian operator and τ is defined as the nonlocal scale coefficient that incorporates the size-dependent small scale factor.