Composite Structures 132 (2015) 885–897

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Optimal design of symmetrically laminated plates for damping characteristics using lamination parameters

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ARTICLE INFO

Article history:
Available online 4 July 2015

Keywords:
Optimal design
Damping
Composite materials
Lamination parameters
Differential evolution

ABSTRACT

The present paper deals with the damping characteristics of symmetrically laminated plates. First, the effect of laminate configuration on the damping characteristics is investigated for cantilevered laminated plates. To examine the effect of laminate configuration, the concept of specific damping capacity is introduced and the damping characteristics are represented on the lamination parameter plane, where the damped stiffness invariants are newly proposed in this paper. Next, the optimal laminate configurations for the cantilevered laminated plates with maximal damping subjected to the constraints on the natural frequencies are determined by using differential evolution in which lamination parameters are used as intermediate design variables. The relation between the laminate configurations and the damping characteristics is discussed based on the concept of lamination parameters.

1. Introduction

Recently, advanced composite materials have been applied to aerospace structures. Damping is an important parameter related to the study of dynamic behavior of fiber reinforced composite structures. It is well known that the flexural vibration of flexible structural components may cause the fatigue failures, and the passive vibration suppression with material damping can be effective. Therefore, structural design considering vibrational damping properties is an indispensable technology for improving vibration characteristics of structures. So far, a lot of research has been carried out from the viewpoints of predicting damping capacities at macro-mechanical level and its optimization [1]. Adams and Bacon [2] proposed the damped element model based on strain energy approach for predicting the damping capacity of narrow, angle-ply laminated plates. Lin et al. [3] have investigated the prediction of the specific damping capacity of anisotropic laminated plates by using the finite element method and the extended damped element model in order to account for transverse shear deformation and rotary inertia effects. Optimal design of composite structures with respect to multiple design criteria including damping has also been examined. Kam and Chang [4] have examined the lay-up optimization of thick laminated plates to maximize the specific damping capacity of the plates. They have studied the effects of length-to-thickness ratio, aspect ratio and ply number on the optimal results. Kam and Chang [5] and Lee et al. [6] have examined the minimal weight design of laminated plates considering natural frequency, specific damping capacity and static deflection.

To the author's best knowledge, the limitation of current research in this field is that the majority of the previous studies were focused on the investigation of the effect of fiber orientation on the damping characteristics under a given ply number, the effects of principal-axis stiffness and bending-torsional coupling on the damping characteristics have not been studied.

With the previous background in mind, the objective of the present research is twofold: (1) To clarify the damping characteristics of symmetrically laminated plates through the numerical results; (2) to propose a lay-up optimization method of symmetrically laminated plates for the damping characteristics. First, in this paper, the effect of laminate configuration on the damping characteristics is investigated for symmetrically laminated plates. To examine the effect of laminate configuration, the concept of specific damping capacity is introduced and the damping characteristics are represented on the lamination parameter plane [7,8], instead of using composite fiber orientation angle. For enabling discussion based on the concept of lamination parameters, we newly propose the damped stiffness invariants in this paper. Next, the optimal laminate configurations for the symmetrically laminated plates with maximal damping subjected to the constraints on the natural frequencies are determined by using differential evolution in which

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http://dx.doi.org/10.1016/j.compstruct.2015.06.076
0263-8223/

lamination parameters are used as intermediate design variables, instead of ply fiber orientation angles and ply thicknesses.

2. Damping characteristics of symmetrically laminated plates

2.1. Fundamental equations

The specific damping capacity of the ith vibration mode Ψ_i associated with the flexural response of cantilevered laminated plate shown in Fig. 1 is defined as [2,9]

$$\Psi_i = \frac{\Delta Z_i}{Z_i} = \frac{\Delta Z_{1,i}}{Z_i} + \frac{\Delta Z_{2,i}}{Z_i} + \frac{\Delta Z_{12,i}}{Z_i} = \Psi_{1,i} + \Psi_{2,i} + \Psi_{12,i}, \quad (1)$$

where the modal strain energy of the ith vibration mode stored in the plate and the dissipated energy of the ith vibration mode during a stress cycle are denoted by Z_i and ΔZ_i, respectively. The energy dissipations and the specific damping capacities due to individual stress components σ_1, σ_2 and τ_{12}, are denoted by $\Delta Z_{1,i}$, $\Delta Z_{2,i}$ and $\Delta Z_{12,i}$, and $\Psi_{1,i}, \Psi_{2,i}$ and $\Psi_{12,i}$, respectively.

In classical lamination theory, the constitutive equation of symmetrically laminated plate can be described as

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & 0 \\ 0 & D \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \kappa \end{Bmatrix}, \quad (2)$$

where the generalized stress components are: $N = \{N_x, N_y, N_{xy}\}^T$, $M = \{M_x, M_y, M_{xy}\}^T$, and the generalized strain components are: $\varepsilon = \{\varepsilon_x, \varepsilon_y, \varepsilon_{xy}\}^T$, $\kappa = \{\kappa_x, \kappa_y, \kappa_{xy}\}^T$, respectively. The in-plane and out-of-plane stiffnesses are denoted by A_{ij} $(i,j = 1,2,6)$ and D_{ij} $(i,j = 1,2,6)$, respectively. In case of symmetrically laminated plates chosen in a practical usage, the in-plane and out-of-plane problems are separated each other, and the out-of-plane problem is dominated by only the out-of-plane stiffness components D_{ij}.

The modal strain energy of the ith vibration mode is

$$Z_i = \frac{1}{2}\int_S \{\kappa_i\}^T [D] \{\kappa_i\} dS, \quad (3)$$

where the curvature vector of the ith vibration mode is denoted by κ_i, and the plate area is denoted by S. The out-of-plane stiffness components D_{ij} in the modal strain energy Z_i can be described by using the stiffness invariants U_i $(i = 1, \ldots, 5)$ and the out-of-plane lamination parameters ξ_i $(i = 9, 10, 11, 12)$ as follows [7]:

$$\begin{Bmatrix} D_{11} \\ D_{22} \\ D_{12} \\ D_{66} \\ D_{16} \\ D_{26} \end{Bmatrix} = \frac{h^3}{12} \begin{bmatrix} 1 & \xi_9 & \xi_{10} & 0 & 0 \\ 1 & -\xi_9 & \xi_{10} & 0 & 0 \\ 0 & 0 & -\xi_{10} & 1 & 0 \\ 0 & 0 & -\xi_{10} & 0 & 1 \\ 0 & \frac{\xi_{11}}{2} & \xi_{12} & 0 & 0 \\ 0 & \frac{\xi_{11}}{2} & -\xi_{12} & 0 & 0 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{Bmatrix}, \quad (4)$$

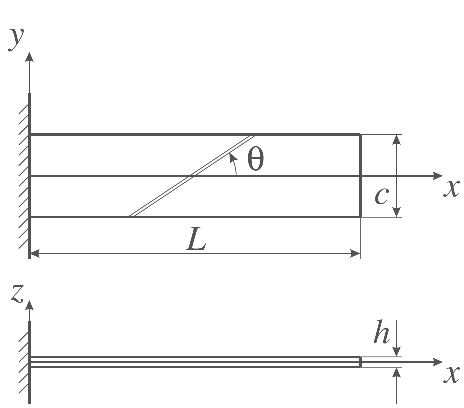

Fig. 1. Cantilevered laminated plate.

where

$$\begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{Bmatrix} = \begin{bmatrix} \frac{3}{8} & \frac{3}{8} & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ \frac{1}{8} & \frac{1}{8} & -\frac{1}{4} & -\frac{1}{2} \\ \frac{1}{8} & \frac{1}{8} & \frac{3}{4} & -\frac{1}{2} \\ \frac{1}{8} & \frac{1}{8} & -\frac{1}{4} & \frac{1}{2} \end{bmatrix} \begin{Bmatrix} Q_{11} \\ Q_{22} \\ Q_{12} \\ Q_{66} \end{Bmatrix}. \quad (5)$$

The thickness of the plate is denoted by h, and the on-axis stiffness components are denoted by Q_{ij} $(i,j = 1,2,6)$. By defining the non-dimensional coordinate along the thickness denoted by $u(= z/(h/2))$, lamination parameters can be expressed as the following forms, using a symmetric condition at the mid-plane [7]:

$$\begin{aligned} \xi_9 &= 3\int_0^1 \cos 2\theta(u) u^2 du, \xi_{10} = 3\int_0^1 \cos 4\theta(u) u^2 du, \\ \xi_{11} &= 3\int_0^1 \sin 2\theta(u) u^2 du, \xi_{12} = 3\int_0^1 \sin 4\theta(u) u^2 du, \end{aligned} \quad (6)$$

where a distribution function of fiber orientation angles through the thickness is denoted by $\theta\ (u)$. Lamination parameters are functionals of the function $\theta\ (u)$ as shown in Eq. (6). When the layer angle distribution is specified through the thickness, the lamination parameters can be determined uniquely from Eq. (6), and the out-of-plane stiffness components can be determined from Eq. (4).

The dissipated energy of the ith vibration mode is

$$\Delta Z_i = \frac{1}{2}\int_S \{\kappa_i\}^T [D_d] \{\kappa_i\} dS. \quad (7)$$

The out-of-plane damped stiffness components $D_{d,ij}$ in the dissipated energy ΔZ_i can be described as follows:

$$D_{d,ij} = 2\int_0^{\frac{h}{2}} R_{ij}(z) z^2 dz \ (\mathrm{i,j} = 1,2,6), \quad (8)$$

where

$$\begin{bmatrix} R_{11} & R_{12} & R_{16} \\ R_{21} & R_{22} & R_{26} \\ R_{61} & R_{62} & R_{66} \end{bmatrix} = \begin{bmatrix} m^2 & n^2 & -2mn \\ n^2 & m^2 & 2mn \\ mn & -mn & m^2 - n^2 \end{bmatrix} \begin{bmatrix} \psi_L & 0 & 0 \\ 0 & \psi_T & 0 \\ 0 & 0 & \psi_{LT} \end{bmatrix} \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} m^2 & n^2 & mn \\ n^2 & m^2 & -mn \\ -2mn & 2mn & m^2 - n^2 \end{bmatrix}, \quad (9)$$

or

$$[R] = \left([T]^{-1}\right)^T [\psi][Q][T]^{-1}, \quad (10)$$

and $m\ (z) = \cos\theta\ (z)$ and $n\ (z) = \sin\theta\ (z)$ in the transformation matrix of strain T. The longitudinal, transverse and longitudinal shear specific damping capacities of a unidirectional lamina are denoted by ψ_L, ψ_T and ψ_{LT}, respectively, which are considered as constant in this paper.

We can develop a multiple angle formulation for the components R_{ij} in place of power functions as follows:

$$\begin{aligned} R_{11} &= m^4\psi_L Q_{11} + m^2n^2(\psi_L + \psi_T)Q_{12} + n^4\psi_T Q_{22} + 4m^2n^2\psi_{LT}Q_{66} \\ &= \left(\frac{3}{8} + \frac{1}{2}\cos 2\theta + \frac{1}{8}\cos 4\theta\right)\psi_L Q_{11} + \left(\frac{1}{8} - \frac{1}{8}\cos 4\theta\right)(\psi_L + \psi_T)Q_{12} \\ &\quad + \left(\frac{3}{8} - \frac{1}{2}\cos 2\theta + \frac{1}{8}\cos 4\theta\right)\psi_T Q_{22} + \left(\frac{1}{2} - \frac{1}{2}\cos 4\theta\right)\psi_{LT}Q_{66} \\ &= \left(\frac{3}{8}\psi_L Q_{11} + \frac{1}{8}(\psi_L + \psi_T)Q_{12} + \frac{3}{8}\psi_T Q_{22} + \frac{1}{2}\psi_{LT}Q_{66}\right) \\ &\quad + \left(\frac{1}{2}\psi_L Q_{11} - \frac{1}{2}\psi_T Q_{22}\right)\cos 2\theta \\ &\quad + \left(\frac{1}{8}\psi_L Q_{11} - \frac{1}{8}(\psi_L + \psi_T)Q_{12} + \frac{1}{8}\psi_T Q_{22} - \frac{1}{2}\psi_{LT}Q_{66}\right)\cos 4\theta, \quad (11.a) \end{aligned}$$