Composite Structures 132 (2015) 974–983

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Large deflection analysis of FG-CNT reinforced composite skew plates resting on Pasternak foundations using an element-free approach

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ARTICLE INFO

Article history:
Available online 14 July 2015

Keywords:
Large deflection
Carbon nanotube
Composite
Skew plates
IMLS-Ritz method

ABSTRACT

The first known geometrically nonlinear large deformation analysis of functionally graded carbon nanotube (FG-CNT) reinforced composite skew plates resting on Pasternak foundations is presented. The skew plate studied is of moderate thickness and, hence, the first-order shear deformation theory (FSDT) and von Kármán assumption are adopted to take care of the transverse shear strains, rotary inertia and moderate rotations. The element-free IMLS-Ritz method is employed in the present analysis. Parametric studies are conducted to examine the effects of CNT content by volume, elastic foundation, skew angle, plate width-to-thickness ratio, plate aspect ratio and boundary conditions on the nonlinear responses of the FG-CNT reinforced composite skew plates. The results of the present study are obtained for simplified cases so that comparison studies can be made with the values reported in the literature.

1. Introduction

Functionally graded materials (FGMs) are specially designed with their material properties spatially varying according to a certain pattern of distribution in their constituent parts, offering a wealth of applications within civil, ocean, marine, nuclear and aerospace engineering. Many studies have been conducted to explore the usage of FGMs in different fields [1] since their introduction [2,3]. Followed the concept of FGMs, the FG-CNT reinforced composite was proposed which follows the FG pattern of reinforcement, with the CNTs uniaxially aligned in the axial direction so that its material properties are graded in the thickness direction [4].

Recently, the FG-CNT reinforced composite has been explored as an advanced material to be embedded in beams, plates or shells, forming useful structural components. In general, this problem can be analyzed by using numerical approaches [5–9]. Using a two-step perturbation technique, Shen [10] presented a nonlinear bending analysis of FG-CNT reinforced composite rectangular plates in thermal environments. Based on a similar approach, postbuckling analyses of FG-CNT reinforced composite cylindrical shells subjected to axial compression and lateral pressure in thermal environments were undertaken by Shen [11,12]. A similar analysis of FG nanocomposite rectangular plates subjected to in-plane temperature variation was conducted by Shen and Zhang [13]. Lei et al. [14] have carried out a nonlinear analysis of FG-CNT reinforced composite rectangular plates using the element-free kp-Ritz method. The geometrically nonlinear large deflection behavior of FG-CNT reinforced composite cylindrical panels under uniform point transverse mechanical loading was studied by Zhang et al. [15]. Shen and Xiang [16] studied the nonlinear bending behaviors of nanotube-reinforced composite cylindrical panels resting on elastic foundations in thermal environments.

Apart from the above studies, the authors have found that no other work has been conducted in relation to the nonlinear analysis of FG-CNT reinforced composite structures. Moreover, to the best of the authors' knowledge, no existing research has been reported on the nonlinear analysis of FG-CNT reinforced composite plate in the skew domain. With the limited literature in mind, the primary purpose of this study is to present a geometrically nonlinear large deformation analysis of FG-CNT reinforced composite skew plates. Recently, meshfree methods are raised for their advantages in computational efficiency compared with finite element methods, especially for nonlinear problems [17]. The analysis is carried out using the element-free IMLS-Ritz method [18–22]. The nonlinear bending formulation is derived based on the first-order shear deformation plate theory and the von Kármán assumption accounting for transverse shear strains, rotary inertia and moderate rotations. Two kinds of CNT reinforced composite

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http://dx.doi.org/10.1016/j.compstruct.2015.07.017
0263-8223/

plates are considered, namely the uniformly distributed and functionally graded distributions of the CNT reinforcement, so that the material properties of FG-CNT reinforced composites are graded in the thickness direction. The results of the present study for simplified cases which make possible verification with the solutions reported in the literature.

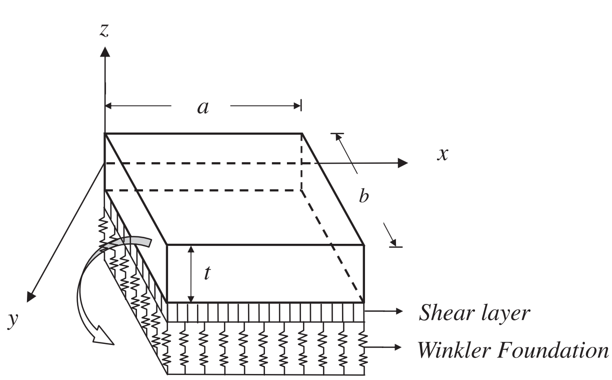
In this paper, detailed parametric studies are carried out to investigate the effects of CNT content by volume, elastic foundation, skew angle, plate width-to-thickness ratio, plate aspect ratio and boundary conditions on the nonlinear responses of FG-CNT reinforced composite skew plates.

2. Material properties of FG-CNT reinforced composite skew plates

Consider an FG-CNT reinforced composite plate resting on Pasternak foundations with length a, width b and thickness t, as shown in Fig. 1. Four types of skew FG-CNT reinforced composite plates are considered. The CNTs are assumed uniaxially aligned, that is UD represents uniform distribution and FG-V, FG-O and FG-X denote the other three types of functionally graded distributions of CNTs. According to the distributions of uniaxially aligned SWCNTs, CNT content by volume is V_{CNT} expressed as

$$V_{CNT}(z) = \begin{cases} V^*_{CNT} & \text{(UD)} \\ \left(1+\frac{2z}{t}\right)V^*_{CNT} & \text{(FG-V)} \\ 2\left(1-\frac{2|z|}{t}\right)V^*_{CNT} & \text{(FG-O)} \\ 2\left(\frac{2|z|}{t}\right)V^*_{CNT} & \text{(FG-X)} \end{cases}, \tag{1}$$

where UD represents the uniform distribution. For the FG-V type, the top surface of the FG-CNT reinforced composite plate is CNT-rich. In the case of FG-O, the mid-plane of the FG-CNT reinforced composite plate is CNT-rich and, for FG-X, both the top and bottom surfaces of the FG-CNT reinforced composite plate are CNT-rich, and



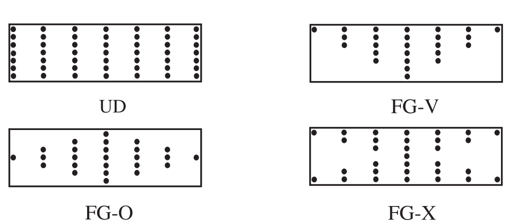


Fig. 1. Configurations of FG-CNT reinforced composite skew plates resting on elastic foundations: (a) UD CNT reinforced composite plate; (b) FG-V CNT reinforced composite plate; (c) FG-O CNT reinforced composite plate; and (d) FG-X CNT reinforced composite plate.

$$V^*_{CNT} = \frac{w_{CNT}}{w_{CNT} + (\rho^{CNT}/\rho^m) - (\rho^{CNT}/\rho^m)w_{CNT}}, \tag{2}$$

in which w_{CNT} is the mass fraction of CNTs and ρ^m and ρ^{CNT} are densities of the matrix and CNTs, respectively. The overall CNT content by volume of UD-CNT reinforced composite plate, and those of the other three types of FG-CNT reinforced composite plates, are the same, which means that the four types of FG-CNT reinforced composite plates have the same mass and volume as the CNTs. In this paper, an equivalent continuum model based on the Eshelby–Mori–Tanaka approach is employed to predict the properties of FG-CNT reinforced composites. For two-phase composites, the effective elastic module tensor $\mathbf{L}$ of FG-CNT reinforced composites can be expressed as follows, according to Benveniste's revision

$$\mathbf{L} = \mathbf{L}_m + V_{CNT}\langle(\mathbf{L}_{CNT} - \mathbf{L}_m)\cdot\mathbf{A}\rangle \cdot [V_m\mathbf{I} + V_{CNT}\langle\mathbf{A}\rangle]^{-1}, \tag{3}$$

where $\mathbf{L}_m$ and $\mathbf{L}_{CNT}$ are stiffness tensors of the matrix and CNT, respectively, and $\mathbf{I}$ is the fourth-order unit tensor. The angle brackets represent an average overall possible orientation of the inclusions. $\mathbf{A}$ is the dilute mechanical strain concentration tensor, and is written as

$$\mathbf{A} = \left[\mathbf{I} + \mathbf{S}\cdot\mathbf{L}_m^{-1}\cdot(\mathbf{L}_{CNT} - \mathbf{L}_m)\right]^{-1}, \tag{4}$$

where $\mathbf{S}$ is the fourth-order Eshelby tensor and is well defined for cylindrical inclusions by Mura [23].

3. Theoretical formulations

3.1. Total potential energy functional

The total potential energy functional of the plate can be expressed as

$$\Pi = U_\varepsilon + V - W_e, \tag{5}$$

in which U_ε is the strain energy of the FG-CNT reinforced composite skew plate, V denotes the stain energy due to Pasternak foundations and W_e is the external work.

The formulation for the large deflection analysis of this plate is derived based on the FSDT. The displacement field expressed on the orthogonal co-ordinates (x, y) is in the following form

$$u(x,y,z) = u_0(x,y) + z\theta_x(x,y), \tag{6}$$

$$v(x,y,z) = v_0(x,y) + z\theta_y(x,y), \tag{7}$$

$$w(x,y,z) = w_0(x,y), \tag{8}$$

where (u, v, w) are the displacements of a generic point (x, y, z) in the FG-CNT reinforced composites plate and (u_0, v_0, w_0) represent the displacement projections on the mid-plane. The transverse normal rotations about the positive y and negative x axes are given by

$$\theta_x = \frac{\partial u}{\partial z}, \quad \theta_y = \frac{\partial v}{\partial z}. \tag{9}$$

Based on the above displacement field, the strain components at a generic point are expressed as

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \boldsymbol{\varepsilon}_0 + z\boldsymbol{\kappa}, \quad \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \boldsymbol{\gamma}_0, \tag{10}$$

where

$$\varepsilon_0 = \varepsilon_{0L} + \varepsilon_{0N} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix} + \begin{Bmatrix} \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 \\ \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 \\ \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y} \end{Bmatrix}, \tag{11}$$