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Exploiting symmetries in solid-to-shell homogenization, with application to periodic pin-reinforced sandwich structures

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ABSTRACT

In this paper, a novel set of Periodic Boundary Conditions named Multiscale Periodic Boundary Conditions (MPBCs) that apply to *reduced* Unit Cells (rUCs) and enable the two-scale (solid-to-shell) numerical homogenization of periodic structures, including their bending and twisting response, is presented and implemented in an FE code. Reduced Unit Cells are domains smaller than the Unit Cells (UCs), obtained by exploiting the internal symmetries of the UCs. When applied to the solid-to-shell homogenization of a sandwich structure with unequal skins, the MPBCs enable the computation of all terms of the fully-populated **ABD** matrix with negligible error, of the order of machine precision. Furthermore, using the MPBCs it is possible to correctly simulate the mechanical response of periodic structures using rUCs (retrieving the same results as if conventional UCs were used), thus enabling a significant reduction of both modelling/meshing and analysis CPU times. The results of these analyses demonstrate the relevance of the proposed approach for an efficient multiscale modelling of periodic materials and structures.

1. Introduction

For a faster structural design of large composite components, the numerical analysis of their mechanical response requires different parts of the structure to be modelled at multiple length/time-scales [1]. In addition, due to the heterogeneous microstructure of composite materials, the applicability of conventional FE analyses based on 3D (solid) models is limited to small components; 2D (shell) FE models with equivalent homogenized properties are preferable for larger components.

Within this framework, numerical homogenization of periodic structures and materials represents a powerful numerical tool. The latter commonly involves the analysis of Unit Cell (UC) models in which the structure is explicitly resolved at the lowest length-scale; several studies on the existence and size of suitable UCs are available in literature [2,3]. Furthermore, numerous works concerning the correct application of Periodic Boundary Conditions (PBCs) to representative UCs can be found, e.g. [4–7] and references therein.

However, for several practical cases, e.g. textile composites and pin-reinforced composite sandwich structures, the topological complexity of the representative UCs may lead to unaffordable modelling/meshing and analysis CPU times. Therefore, the internal symmetries of the UCs should, whenever they exist, be exploited to reduce the analysis domain, thus enabling a reduction of both the modelling/meshing and analysis CPU times.

Domains smaller than the UCs, obtained by exploiting the internal symmetries of the latter, are referred to as *reduced* Unit Cells (rUCs). Several works can be found discussing the determination of suitable rUCs (and corresponding appropriate PBCs) for UD composites [8], particle-reinforced composites [9] and textile composites [10–12].

The PBCs proposed in these studies allow the determination of the homogenized 3D elasticity tensor of the investigated structure. Nevertheless, for the numerical analysis of large components using equivalent shell models, it is of interest to determine the homogenized shell constitutive response of the structure using high-fidelity three-dimensional UCs or, preferably, rUCs.

To address this, we present a novel set of PBCs named Multiscale Periodic Boundary Conditions (MPBCs), that represents the first set of PBCs that apply to rUCs and enable the direct two-scale (solid-to-shell) numerical homogenization of periodic structures, including their bending and twisting response.

The proposed MPBCs are formulated in Section 2.2 while their complete derivation is provided in Appendix A. Details on their use within the context of a solid-to-shell homogenization and on their FE implementation are provided in Sections 2.3 and 3, respectively. The MPCBs are applied to the solid-to-shell homogenization of a sandwich structure with unequal skins in Section 4.1 and to the analysis of the mechanical response of a periodic sandwich

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structure with unequal skins and pin-reinforced core in Section 4.2. Conclusions are drawn in Section 5.

2. Theory

2.1. Problem formulation

Consider a three-dimensional deformable body $\mathcal{B}_3$ occupying the domain $\mathfrak{D}_3 \in \mathbb{R}^3$, as shown in Fig. 1, and assume that one of the dimensions of $\mathfrak{D}_3$, designated as the thickness (denoted as h), is much smaller than the other two. Furthermore, let $\mathfrak{D}_3$ be periodic in the plane normal to the thickness direction. For reasons of numerical efficiency, the macroscopic mechanical response of $\mathcal{B}_3$ is more conveniently simulated by means of an equivalent shell model with homogenized properties contained in $\mathfrak{D}_2 \in \mathbb{R}^2$. The corresponding shell constitutive response can be expressed by the relation $\mathbf{R} = \mathbf{K}\xi_\circ$, where $\mathbf{R}$ is the vector of resultant forces/moments per unit-length acting on the structure, $\xi_\circ$ is the resultant mid-surface membrane strains/curvatures vector and the $\mathbf{K}$ matrix (referred to, in lamination theory, as the **ABD** matrix) contains the equivalent shell stiffness terms.

Therefore, the accurate determination of the entire $\mathbf{K}$ matrix, including all bending and twisting terms, as well as the shear-extension, bending-extension and bending-twisting coupling terms, is of paramount importance for the use of equivalent shell models of large composite components.

2.2. Multiscale Periodic Boundary Conditions (MPBCs)

The homogenized 2D response of a three-dimensional periodic structure can be evaluated through the analysis of three-dimensional UCs/rUCs modelled at the meso-scale, provided the appropriate PBCs are applied to its boundaries. These PBCs, referred to in the following as Multiscale PBCs (MPBCs), are prescribed such that the three-dimensional UC/rUC behaves as it were contained in an infinite body $\mathcal{B}_3$ (in domain $\mathfrak{D}_3$) whose macroscopic response can be analysed using shell theory (in domain $\mathfrak{D}_2$).

Assume that domain $\mathfrak{D}_3$ can be reconstructed from the tessellation of repeating UCs, as schematically shown in Fig. 2; in addition, let these UCs have internal symmetries, which if exploited lead to the definition of rUCs (see Fig. 2). Moreover, let $\mathfrak{s} \in \mathfrak{D}_3$ and $\hat{\mathfrak{s}} \in \mathfrak{D}_3$ be two adjacent subdomains satisfying both the physical and loading equivalence (Eqs. (A.1) and (A.2)) and let $\mathfrak{s}$ be a suitable rUC as defined in Appendix A.2. Furthermore, denote with $[O, \mathbf{e}_i]_\mathfrak{s}$ and $[\hat{O}, \hat{\mathbf{e}}_i]_{\hat{\mathfrak{s}}}$ the Local Coordinate Systems (LCSs) of subdomains $\mathfrak{s}$ and $\hat{\mathfrak{s}}$, respectively, and define the two-dimensional symmetry transformation matrix between $[O, \mathbf{e}_i]_\mathfrak{s}$ and $[\hat{O}, \hat{\mathbf{e}}_i]_{\hat{\mathfrak{s}}}$ as $\mathbf{T} = \{T_{ij}\}$ where $T_{ii} = \dfrac{\partial \mathbf{e}_i}{\partial \hat{\mathbf{e}}_i}$ with $i \in \{1, 2\}$ and $T_{ij} = 0$ if $i \neq j$. In the following, the coordinate values along the direction $\mathbf{e}_i$ are indicated as x_i. Consider now two equivalent points $A \in \mathfrak{s}$ and $\hat{A} \in \hat{\mathfrak{s}}$ (see Eq. (A.1)); if A is at the boundary of $\mathfrak{s}$, i.e. $A \in \partial\mathfrak{s}$, then also $\hat{A} \in \partial\hat{\mathfrak{s}}$ (see Fig. 2). Under these premises, the proposed MPBCs can be expressed as

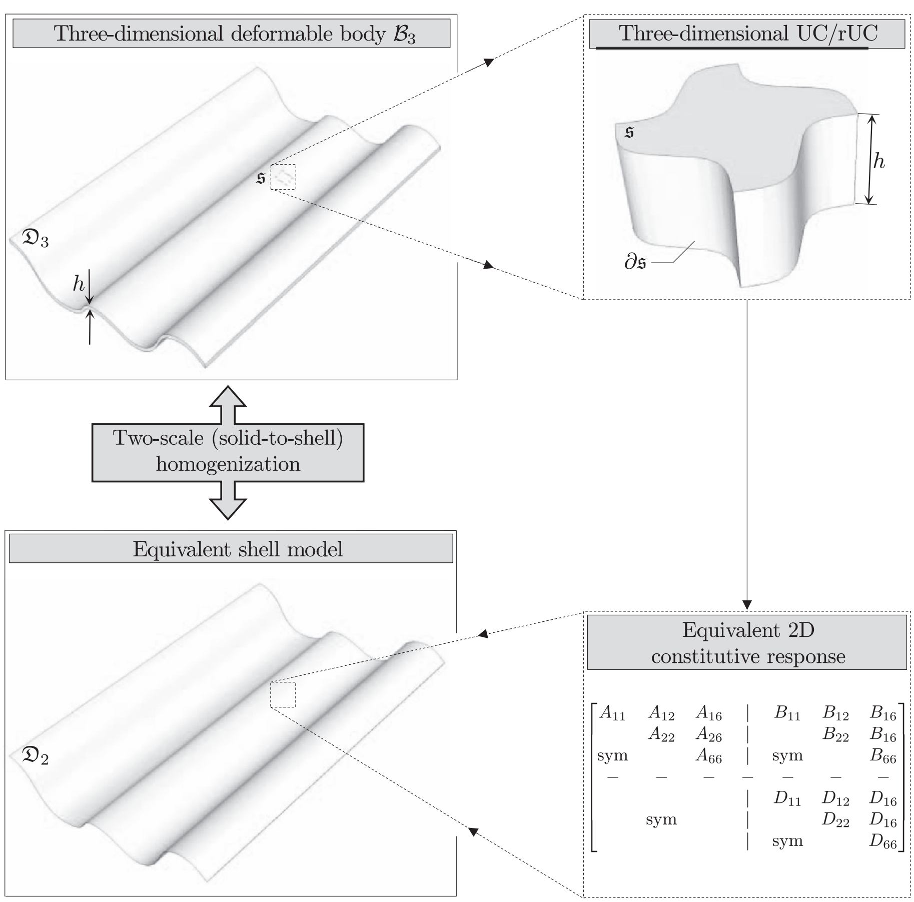

Fig. 1. Problem formulation. The macroscopic response of the periodic domain $\mathfrak{D}_3$ can be simulated by means of an equivalent shell model, provided the equivalent 2D constitutive response of a representative three-dimensional UC/rUC (subdomain $\mathfrak{s}$) is correctly determined.