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Three-dimensional thermal shock plate problem within the framework of different thermoelasticity theories

Ashraf M. Zenkour *

Department of Mathematics, Faculty of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia
Department of Mathematics, Faculty of Science, Kafrelsheikh University, Kafr El-Sheikh 33516, Egypt

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ABSTRACT

The exact three-dimensional solutions of the temperature, displacements and stresses of thermal shock plate problem are presented. The bottom surface of the plate is thermally isolated while the upper one is subjected to a thermal shock. A unified generalized thermoelasticity theory for the transient thermal shock plate problem in the context of Green and Lindsay, Lord and Shulman, and coupled thermoelasticity theories is presented. The variations along the longitudinal and thickness directions of all fields are investigated. Some comparisons have been shown graphically to estimate the effects of different parameters on all the studied fields. The analytical general solution is applied to the present plate using the normal mode analysis. A comparison between different theories is presented and suitable conclusions are made.

1. Introduction

Many generalized theories of thermoelasticity have been developed in the literature to study the behavior of thermoelastic structures. The most famous theories are mentioned such as the theory of coupled thermoelasticity (CTE) [1], the Lord and Shulman (L–S) theory [2], the Green and Lindsay (G–L) theory [3] as well as the Green and Naghdi (G–N) theory [4–6]. To the author's best knowledge, only a few authors have presented the exact three-dimensional solution of the generalized thermoelastic plate problem up to present time. Most authors used the classical theory for thin plates as well as one of the generalized thermoelasticity theories.

Many thermoelastic problems of thermal shock are presented in the literature. Mukherjee and Sinha [7] have examined the coupled dynamic thermoelastic response of a fibrous composite plate exposed to a thermal shock. He et al. [8] have used the G–L generalized thermoelasticity theory with two relaxation times to deal with a thermoelastic, piezoelectric coupled 2-D thermal shock problem of a thick piezoelectric plate of infinite extent by means of the hybrid Laplace transform-finite element method. He et al. [9] have considered a generalized electromagneto-thermoelastic coupled problem of a perfectly conducting half-space solid subjected to a thermal shock on its surface based on the L–S theory. Daneshjo and Ramezani [10] have proposed a new mixed finite element formulation to analyze transient coupled model of dynamic thermoelasticity for laminated composite and homogeneous isotropic plates.

Zenkour and Abbas [11] have presented the G–N theory to study the influence of the magnetic field for the thermal shock problem of a fiber-reinforced anisotropic half-space. Sherief et al. [12] have considered a half-space of a thermoelastic material subjected to a thermal shock. Liu et al. [13] have presented the transient thermal dynamic analysis of stationary cracks in functionally graded piezoelectric materials based on the extended finite element method. Ezzat and Youssef [14] have presented a 3-D model of the generalized thermoelasticity with one relaxation time applied to a specific problem of a half-space subjected to thermal shock and traction free surface using the Laplace and double Fourier transforms techniques. Kar and Kanoria [15] have determined the thermo-elastic interaction due to step input of temperature on the boundaries of a functionally graded orthotropic hollow sphere in the context of linear theories of generalized thermo-elasticity. Hosseini–Tehrani and Hosseini–Godarzi [16] have presented a boundary element formulation for the crack analysis utilizing the L–S theory of thermoelasticity for a body exposed to a thermal shock on its boundary. Wang et al. [17] have presented a unified generalized thermoelasticity solution for the transient thermal shock problem in the context of three different generalized theories of the coupled thermoelasticity. Das et al.

* Address: Department of Mathematics, Faculty of Science, Kafrelsheikh University, Kafr El-Sheikh 33516, Egypt.
E-mail address: zenkour@gmail.com

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[18] have dealt with a problem of magneto-thermoelastic interactions in a transversely isotropic hollow cylinder due to thermal shock in the context of three-phase-lag theory of generalized thermoelasticity.

The present article is concerned with the three-dimensional transient generalized thermoelastic problem for a plate subjected to a thermal shock acting on its upper surface while its bottom surface is thermally insulated. The G–L theory [19] of generalized thermoelasticity is constructed and other known thermoelastic theories such as CTE and L–S [20,21] may be also considered. The analytical general solution is applied to the present plate using the normal mode analysis [22–25]. Numerical results showing the thermoelastic dynamic responses of the field quantities through the longitudinal and thickness directions of the plate are presented. The effects of the side-to-thickness ratio, aspect ratio and time parameters are also investigated.

2. Formulation of the problem

Let us consider a solid rectangular plate of dimensions $(a \times b \times h)$ as shown in Fig. 1. The considered homogenous, isotropic plate is initially un-deformed and at uniform temperature T_0 in Cartesian coordinate system $Ox_1x_2x_3$. The basic governing questions of motion and heat conduction in the context of generalized (non-Fourier) thermoelasticity for displacements $u_i(x_i, t)$ in the absence of body forces should be considered.

The Duhamel–Neumann thermoelastic law for an isotropic plate is written in the form

$$\sigma_{ij} = 2\mu e_{ij} + \lambda e \delta_{ij} - \gamma\left(\theta + \tau_1\dot{\theta}\right)\delta_{ij}, \tag{1}$$

where λ and μ are Lame's coefficients, $\gamma = (3\lambda + 2\mu)\alpha$ is the stress–temperature modulus in which α is the linear thermal expansion coefficient, $\theta = T - T_0$ is the temperature increment of the resonator, in which $T(x_i)$ is the absolute temperature distribution and T_0 is the environmental reference temperature, δ_{ij} is the Kronecker's delta function, and τ_1 is the first relaxation time of Green and Lindsay's theory. The strain–displacement relations e_{ij} and the volumetric strain e are taken in the following linear form

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \quad e = u_{k,k}. \tag{2}$$

It is to be noted that, a comma followed by index j denotes partial differentiation with respect to the position x_j of a material particle. A superimposed dot indicates partial derivative with respect to time t.

In the absence of body forces and internal heat generation, the heat conduction equation will be in the form

$$K\nabla^2\theta = \rho C^e\left(\dot{\theta} + \tau_2\ddot{\theta}\right) + \gamma T_0(\dot{e} + \tau_3\ddot{e}), \tag{3}$$

where K is thermal conductivity coefficient, τ_2 and τ_3 are additional second and third relaxation times, ρ is the material density and C^e is the specific heat per unit mass at constant strain.

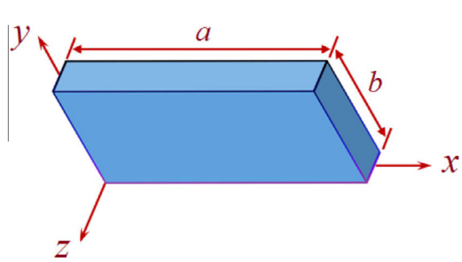

Fig. 1. Schematic diagram of the plate.

It is clear that, by setting $\tau_1 = \tau_2 = \tau_3 = 0$, one gets the field equations for the conventional coupled theory of thermoelasticity (CTE); whereas when $\tau_1 = 0$ and $\tau_2 = \tau_3 \neq 0$, the equations reduce to the Lord and Shulman's theory (L–S) and when $\tau_3 = 0$ and τ_1 and τ_2 are non-vanishing, the generalized thermoelasticity equations are reduced to the Green and Lindsay's theory (G–L).

The governing equations of motion can be presented as

$$\begin{aligned}
&(\lambda+2\mu)u_{1,11} + \mu(u_{1,22}+u_{1,33}) + (\lambda+\mu)(u_{2,2}+u_{3,3})_{,1} - \gamma\left(\theta+\tau_1\dot{\theta}\right)_{,1} = \rho\ddot{u}_1,\\
&(\lambda+2\mu)u_{2,22} + \mu(u_{2,11}+u_{2,33}) + (\lambda+\mu)(u_{1,1}+u_{3,3})_{,2} - \gamma\left(\theta+\tau_1\dot{\theta}\right)_{,2} = \rho\ddot{u}_2,\\
&(\lambda+2\mu)u_{3,33} + \mu(u_{3,11}+u_{3,22}) + (\lambda+\mu)(u_{1,1}+u_{2,2})_{,3} - \gamma\left(\theta+\tau_1\dot{\theta}\right)_{,3} = \rho\ddot{u}_3.
\end{aligned} \tag{4}$$

The cubical dilatation definition is used here to rewrite the above equations in an alternative form as

$$\begin{aligned}
&\mu\nabla^2 u_{1,1} + (\lambda+\mu)e_{,11} - \gamma\left(\theta+\tau_1\dot{\theta}\right)_{,11} = \rho\ddot{u}_{1,1},\\
&\mu\nabla^2 u_{2,2} + (\lambda+\mu)e_{,22} - \gamma\left(\theta+\tau_1\dot{\theta}\right)_{,22} = \rho\ddot{u}_{2,2},\\
&\mu\nabla^2 u_{3,3} + (\lambda+\mu)e_{,33} - \gamma\left(\theta+\tau_1\dot{\theta}\right)_{,33} = \rho\ddot{u}_{3,3}.
\end{aligned} \tag{5}$$

To get the governing equations in more convenient forms, one can introduce the following dimensionless variables

$$\begin{aligned}
&\{\acute{x}_i, \acute{u}_i\} = \eta c\{x_i, u_i\}, \quad \{\acute{t}, \acute{\tau}_i\} = \eta c^2\{t, \tau_i\}, \quad \acute{\theta} = \frac{\gamma\theta}{\lambda+2\mu},\\
&\acute{\sigma}_{ij} = \frac{3}{3\lambda+2\mu}\sigma_{ij},
\end{aligned} \tag{6}$$

where

$$\eta = \frac{\rho C^e}{K}, \quad c^2 = \frac{\lambda+2\mu}{\rho}. \tag{7}$$

Adding the three parts of Eq. (5) and applying the above dimensionless variables, we obtain (dropping the prime for convenience)

$$\nabla^2\left(e - \theta - \tau_1\dot{\theta}\right) = \ddot{e}. \tag{8}$$

In addition, the heat equation and the stress components become

$$\nabla^2\theta = \dot{\theta} + \tau_2\ddot{\theta} + \varepsilon(\dot{e} + \tau_3\ddot{e}), \tag{9}$$

$$\sigma_{ij} = 3\left(\lambda_1 e_{ij} + \lambda_2 e\delta_{ij}\right) - \lambda_3\left(\theta + \tau_1\dot{\theta}\right)\delta_{ij}, \tag{10}$$

where

$$\begin{aligned}
&\lambda_1 = \frac{2\mu}{3\lambda+2\mu}, \quad \lambda_2 = \frac{\lambda}{3\lambda+2\mu}, \quad \lambda_3 = \frac{3(\lambda+2\mu)}{3\lambda+2\mu},\\
&\varepsilon = \frac{\gamma^2 T_0}{\eta K(\lambda+2\mu)}.
\end{aligned} \tag{11}$$

Now, let us consider the invariant stress σ to be the mean value of the principal stresses as

$$\sigma = \frac{1}{3}\sum_{i=1}^{3}\sigma_{ii} = e - \lambda_3\left(\theta + \tau_1\dot{\theta}\right). \tag{12}$$

3. Normal mode analysis

The solution of the considered displacements, temperature, and stresses can be decomposed in terms of normal mode to suit the actual situation of the problem as the following form:

$$\{u_i, e, \theta, \sigma_{ij}\}(x_1, x_2, x_3, t) = \left\{u_i^*(x_3), e^*(x_3), \theta^*(x_3), \sigma_{ij}^*(x_3)\right\}e^{\omega t + i(mx_1 + nx_2)}, \tag{13}$$