Composite Structures 132 (2015) 1085–1093

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Nonlinear deformations of piezoelectric composite beams

Vahid Tajeddini, Anastasia Muliana *

Department of Mechanical Engineering, Texas A&M University, College Station, United States

ARTICLE INFO

Article history:
Available online 2 July 2015

Keywords:
Electro-active composites
Flexible actuators
Folding
Viscoelastic

ABSTRACT

This study presents large deformations of slender elastic and viscoelastic beams with multiple piezoelectric patches attached on their top and bottom surfaces. The slender beams can undergo large in-plane 2-D deformations due to electric fields applied through the piezoelectric patches and mechanical actuations. A nonlinear electro-mechanical constitutive equation is considered for the piezoelectric patches, while linear elastic and viscoelastic constitutive equations are used for the beams. Reissner's finite-deformation beam theory is adopted in formulating the large 2-D deformation, and modified in order to incorporate the deformation due to the electric field input. For an elastic beam, closed form solutions are obtained for the deformations of the beam under electric field actuation, while a nonlinear shooting method is used to analyze the deformation of the beam under both electrical- and mechanical stimuli. For viscoelastic beams, time-dependent deformations of the beams under electric field actuations through the piezoelectric patches are solved numerically. By applying electric fields with different amplitude at different locations of patches, desired deformed shapes in active flexible beams can be achieved, which is useful for analyses and designs of active foldable systems. This study also highlights the effect of viscoelastic materials on the shape changes in foldable electro-active beams.

1. Introduction

Piezoelectric materials, which are considered as smart materials, experience deformations under application of electric fields and produce electric charges when subjected to mechanical loads. These materials are commonly used as sensors and actuators in practical applications. As sensors, they can be used to monitor changes in deformations or strains due to pressures or chemical reactions, such as in oil wells. As actuators, they can be used to control deformations or attain shape changes in active structures by applying electric fields through the piezoelectric materials that are integrated to the structures. There have been several studies on analyzing deformations in elastic beams and plates having distributed piezoelectric patches, which are used in vibration controls, acoustic applications, and noise control systems [1]. For example, by using piezoelectric actuators, dynamic performance of aerospace structures are controlled through increasing damping of the structure and decreasing vibration amplitudes [2]. Most of these studies only consider small deformations in the active structures and relatively small amplitude of electric field is needed for vibration suppression.

Recent development in adaptive foldable (flexible) structures allows for controllable reconfiguration into various shapes, which have many engineering applications, e.g., artificial skins, morphing aircraft, flexible robots for use in hazardous environments, etc. One example of achieving active flexible systems is by integrating piezoelectric ceramics into homogeneous polymeric matrix, termed as electro-active composites, which form lightweight flexible (compliant) active materials. When subjected to external stimuli, flexible/compliant systems generally experience large deformations. There have been analytical and numerical studies presented on simulating large deformations of flexible structures, such as slender beams, subjected to a mechanical stimulus. Most of these studies are available for elastic and homogeneous structures. Commonly used analytical methods for obtaining solutions to large deformations of elastic beams include power series, equivalent systems [3], e.g., pseudo-linear systems, which solve the nonlinear problems by transforming the equations into a set of linear equations, and elliptical integrals [4,5], while the numerical approaches include Runge–Kutta, shooting method and finite element analyses, see Tada and Lee [6], Yang [7] and Chajes [8]. In the deformation analysis of slender beams undergoing large deformations, the slope of the deflected middle axis cannot be neglected in determining the curvature of the beam. Reissner [9] formulated the governing equations for large 2-D displacements and finite strains of elastic beams subjected to mechanical loadings. Later, Irschik and Gerstmayr [10] derived the Reissner's governing equations for originally straight beams based on continuum mechanics. The

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* Corresponding author. Tel.: +1 979 458 3579.
E-mail address: amuliana@tamu.edu (A. Muliana).

http://dx.doi.org/10.1016/j.compstruct.2015.06.041
0263-8223/

governing equations are expressed in the Lagrangian configuration and results in a system of nonlinear differential equations, which can be solved numerically or analytically depending on the prescribed boundary conditions. It is noted that for slender bodies whose dimensions are not of comparable orders of magnitude, the bodies can undergo large deformations, mainly due to large rotations, while the strains in the bodies remain small [11,12].

There have been limited studies on analyzing nonlinear deformations of smart structures. Lagoudas and co-authors [13–15] studied the deformations of flexible rods with embedded shape memory alloys by using shear-lag model in order to approximate the axial forces and moments induced by the actuators. Banerjee et al. [16] presented nonlinear shooting and Adomian decomposition methods in order to obtain solutions to large deformations of cantilever beams under mechanical loadings, which can be extended to predict large deformations in cantilever beams with piezoelectric materials. A linear elastic constitutive model was considered for the cantilever beams. In actuation applications, it is often necessary to apply high electric field inputs to the piezoelectric components in order to obtain large deformations. When subjected to high electric fields, the piezoelectric materials often experience nonlinear electro-mechanical responses, see Tiersten [17]. Tiersten formulated an electro-mechanical constitutive model by considering higher order terms of the electric field in order to describe the nonlinear electro-mechanical coupling behavior of piezoelectric ceramic (PZT-G) materials. A limited number of studies have considered nonlinear electro-mechanical response of piezoelectric materials due to large electric fields [18–20], but for small deformations. In the present study, higher order terms of the electric field are taken into account in the electro-mechanical constitutive relation of smart beams undergoing large deformations.

In many flexible structures, polymers are widely used because of their capability in undergoing large deformations. One of the prominent characteristics of polymers is their time-dependent (viscoelastic) behavior. It might be necessary to consider the time-dependent behaviors in the viscoelastic polymeric structures when non-mechanical stimuli are prescribed in order to obtain shape changes. There are limited studies that address large deformations of viscoelastic beams, e.g., Ya-Peng and Ya-Fei [21], Holden [22], Baranenko [23], Lee [24], Vaz and Caire [25]. Both analytical and numerical approaches for large deformation of the beams with linear viscoelastic constitutive model have been considered. To the best of our knowledge, very limited studies address nonlinear deformation of active viscoelastic structures. Beldica and Hilton [26] studied stress, deformation and failure of a fiber composite beam with piezoelectric layers by considering general nonlinear anisotropic viscoelastic constitutive relations for piezoelectric layers and the beam. For incorporating large deformations they took into account a large rotation in expressing the curvature of the middle axis of the beam. Muliana [27] presented large deformation analyses of viscoelastic polymeric beams by adopting the Reissner finite strain beam theory. Linear and nonlinear viscoelastic constitutive models were considered, and under relatively large external stimuli the linear and nonlinear constitutive models resulted in significantly different responses.

The present study analyzes nonlinear deformations of active beams having multiple piezoelectric patches, actuated by electric fields. Linear and nonlinear electro-mechanical constitutive models are incorporated for the piezoelectric patches, while elastic and viscoelastic behaviors are considered for the host beams. The first part presents an analysis of large deformations of a smart elastic slender cantilever beam, i.e., homogeneous elastic beam with arbitrary number of piezoelectric actuator patches. The beam is assumed relatively slender so that the effect of the transverse shear deformations on the lateral deflections of the beam can be neglected. The actuators are placed in the form of pairs of piezoelectric patches, on the top and bottom surfaces of the beam symmetrically with respect to the middle axis of the beam. To induce large deformations and sharp curvatures in the beam, the top and bottom actuators are subjected to large electric fields so that they experience opposite elongation and contraction along the longitudinal axis of the beam. Under such electric field inputs, the beam experiences bending. The governing equations for large deformations in an elastic beam formulated by Reissner [9] are adopted and modified to include the electro-mechanical coupling effect from the piezoelectric patches. It is assumed that the plane that is perpendicular to the longitudinal axis of the undeformed beam remains plane during the deformations, which is a sensible assumption for a slender beam. Analytical solutions of the governing equations are then presented for the deformations of the beams. When the cantilever smart beam is also subjected to mechanical loads, the nonlinear shooting method [16] is used to convert the boundary value problem from the governing equations of the deformations of the beams to an initial value problem, and a 4th-Runge–Kutta method is used to numerically solve the initial value problem. The second part of this study considers deformations of smart viscoelastic thin beams actuated by electric fields. The piezoelectric patches induce bending moments in the homogeneous beam, which is modeled with linear viscoelastic constitutive relations and the nonlinear Reissner kinematic relations are then considered to obtain the time-dependent deformations of the beam.

The outline of the paper is as follows: Section 2 presents the governing equations for large deformations of 2-D elastic beams subject to mechanical and electrical stimuli, followed by a nonlinear shooting method for solving boundary value problems. Section 3 discusses time-dependent analyses of viscoelastic beams with piezoelectric patches. Section 4 presents several boundary values problems for both elastic and viscoelastic beams with piezoelectric patches. The present study can be used for preliminary design of active compliant systems. Several parameters, such as number and location of piezoelectric patches and magnitude of electric fields are varied in order to control the deformations of the beam and obtain desired shapes. Section 5 summarizes the present work.

2. Governing equations for active elastic beams

Consider an originally straight cantilever beam under electric field inputs applied to the pair of piezoelectric patches, as shown in Fig. 1. In order to induce bending in the beam, the electric fields on the top and bottom patches should be applied such that one would cause elongation along the longitudinal axis of the beam while the other would cause contraction. In many practical applications, thin piezoelectric patches are considered and the electric fields are applied through the thickness of the patches.

Crawley and de Luis [28] showed that when the actuator is perfectly bonded to the surface of its host structure, shear force is transferred from the excited actuator to the host structure near the edges of the patch. It is assumed that the piezoelectric patch has significantly smaller length compared to the length of the beam and also it is sufficiently thin so that the stress components in the thickness direction are neglected and the distribution of the extensional strain through the thickness of the actuator is assumed uniform. The transverse surfaces of the beam is assumed to remain straight during the deformation. Moreover, it is assumed that the center line of the beam is inextensible; and in such case large deformation in the slender beam is due to large rotation gradient, while the strain remains small [11,12]. Therefore, when a small strain is considered and the transverse surfaces of the beam is