Composite Structures 135 (2016) 96–108

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Nonlinear vibrations of a reinforced composite plate with carbon nanotubes

X.Y. Guo [a,b], W. Zhang [a,*]

[a] Beijing Key Laboratory of Nonlinear Vibrations and Strength on Mechanical Structures, College of Mechanical Engineering, Beijing University of Technology, China
[b] Key Laboratory of Dynamics and Control of Flight Vehicle, Ministry of Education, Beijing 100124, China

ARTICLE INFO

Article history:
Available online 18 September 2015

Keywords:
Nonlinear vibrations
Carbon nanotubes
Reinforced composite plate
Chaotic dynamics

ABSTRACT

The nonlinear vibration behaviors of a reinforced composite plate with the carbon nanotubes (CNT) under combined the parametric and forcing excitations are studied in this paper for the first time. Based on the Mori–Tanaka method and the method of calculating the average stress of composite materials, the effective modules for the carbon nanotubes are obtained. The carbon nanotubes are supposed to be long hollow cylindrical structures. The nonlinear partial differential governing equations of motion for the CNT reinforced composite thin plate are derived by using the Reddy's third-order shear deformation plate theory and the geometric nonlinearity of von Karman. The governing equations are discrete to a set of ordinary differential equations by using the Galerkin method. A perturbation method is then employed to obtained the four-dimensional averaged equations of the CNT reinforced composite thin plate under the cases of 1:1 internal resonances. The nonlinear resonant responses of the CNT reinforced composite rectangular thin plate are analyzed under combined the parametric and forcing excitations. The effects of the forcing excitations on the different kinds of the periodic and chaotic motions of the CNT reinforced composite rectangular thin plate are investigated through a comprehensive parametric study.

1. Introduction

As one of the new potential high quality materials, carbon nanotubes introduce a special way to produce advanced fiber reinforced composites. In the past two decades, significant advancements have been achieved in high performance composites, especially in carbon reinforced composites. In aeronautic and astronautic applications, the pursuit of structural materials of light weight and high strength remains an everlasting goal, while carbon nanotubes reinforced polymer composites become very attractive as a promising new generation material due to their outstanding properties. Therefore, it is necessary to study the nonlinear dynamics of this new kind of composite material in order to provide more helpful parameters for the engineering application.

Several investigations on the mechanics of the CNT composite plates are available in the literatures. Qian et al. [1] showed that the stiffness of a resulting composite film can be increased by around 36–42% and the tensile strength by 25% when only 1% (by weight) of CNTs were added in a matrix material. Some articles expressed the interest on mechanical and physical properties of nanotubes composites [2,3]. Kireitseu et al. [4] found that the toughness and stiffness of nanotube reinforced polymer composites was related to functions of the Young's modulus of the nanotubes. Based on the elasticity theory for evaluating effective material properties of CNT reinforced composites, Spitalsky et al. [5] gave the mechanical and electrical properties of nanocomposites of various carbon nanotube contents. Joshi [6] studied the effective material properties for the hexagonal representative volume elements with long and short straight CNTs under the axial loading condition. Wang et al. [7] examined the mechanical behaviors of the PA-6 nanotube fibers reinforced polymethylmethacrylate (PMMA) transparent composites with different processing methods and pointed out that the tensile modulus and the strength of the composites could be increased by nearly 20% if the coaxially electrospun nanofibers were used.

On the analytical researches, Griebel and Hamaekers [8] gave a review of the mechanical reinforcement of polymers using carbon nanotubes. The possibility of using single wall carbon nanotubes (SWCNT) and multi-wall carbon nanotubes (MWCNT) as reinforcing composites has been studied. Coleman et al. [9] analyzed the elastic modulus of the composite structures with CNT polyethylene reinforced and found that the adoption of long nanotubes could

* Corresponding author.
E-mail address: sandyzhang0@yahoo.com (W. Zhang).

http://dx.doi.org/10.1016/j.compstruct.2015.08.063
0263-8223/

lead to a better reinforcement. Song and Youn [10] used the asymptotic expansion homogenization (AEH) method to investigate the elastic properties of composite structures reinforced by carbon nanotubes. In the Refs. [11,12], researchers found that adding small amount of carbon nanotubes in the matrix could improve the mechanical, electrical and thermal properties of polymeric composites. Ke et al. [13] studied the nonlinear free vibrations of functionally graded composite beams reinforced by aligned, straight CNTs. Formica et al. [14] employed an equivalent continuum model to study the vibration behaviors of cantilevered CNT reinforced composite plates according to the Mori–Tanaka scheme. Wang and Shen [15] studied the large amplitude nonlinear vibrations of CNT reinforced composite plates resting on an elastic foundation in thermal environments. Besides, Shen and others [16,17] also studied the buckling behaviors of functionally graded carbon nanotube-reinforced composite plates under different boundary condition. Free vibration analyses of thin-to-moderately thick composite plates reinforced by single-walled carbon nanotubes were presented using the finite element method by Zhu et al. [18]. Wang and Shen [19] presented the analysis on the nonlinear vibration and bending of elastically supported sandwich plates with carbon nanotube reinforced composite face sheets in the thermal environment. Based on the the Euler–Bernoulli beam theory, Rafiee et al. [20] investigated the geometrically nonlinear free and forced oscillations of simply supported single walled carbon nanotubes (SWCNTs) and discussed the nonlinear frequencies of SWCNTs with initial lateral displacement. Yas et al. [21] reported the vibration properties of functionally graded nano composite cylindrical panels reinforced by single-walled carbon nanotubes (SWCNTs) based on the three-dimensional theory of elasticity. Later, Rafiee et al. [22] established nonlinear dynamic stability of initially imperfect piezoelectric functionally graded carbon nanotube reinforced composite (FG-CNTRC) plates under a combined thermal and electrical loadings and interaction of parametric and external resonance.

Recently, the nonlinear vibrations of the different composite structures also been reported. Hao et al. [23] obtained the nonlinear governing equations of a functionally graded material (FGM) plate and analyzed the nonlinear oscillations, bifurcations and chaos of the FGM rectangular plate under certain conditions. Zhang et al. [24–26] analyzed the nonlinear dynamics of composite laminated plates with the transverse and parametric excitations. Guo et al. [27] experimentally and theoretically studied the nonlinear dynamics of a simply supported composite laminated rectangular thin plate with the parametric excitation. The low-frequency large amplitude nonlinear vibrations excited by a high-frequency small amplitude mode were found. Mareishi et al. [28] investigated the nonlinear free vibration of piezoelectric laminated composite beams and the effects of the temperature rise and the volume fraction of the piezoelectric fibers on the nonlinear fundamental natural frequencies of the piezoelectric fiber reinforced composites. Zhang et al. [29] analyzed the frequency–response curves of strongly coupled of the 3D-Kagome truss core sandwich plate subjected to the transverse and the in-plane excitations. The results indicate that there are the hardening and softening nonlinearities in the truss core sandwich plate under the specific resonant case.

In this paper, we focus on the nonlinear vibrations of simply supported symmetric CNT reinforced composite rectangular thin plates subjected to combined in-plane and transverse excitations. The effective modules for the carbon nanotubes are obtained by using the Mori–Tanaka method and the method of calculating the average stress of composite materials. The governing equations of motion for the CNT reinforced composite plate are derived using Hamilton's principle. With the help of the Galerkin discretization, the partial differential governing equations of motion are transformed to the nonlinear ordinary differential equations under combined parametric and forcing excitations. Furthermore, the method of multiple scales is used to obtain four-dimensional averaged equations in the cases of 1:1 internal resonances. The averaged equations are analyzed to obtain the periodic and chaotic motions of the nanotube reinforced composite plate using the Runge–Kutta algorithm. The influence of the excitation on the nonlinear vibration responses is studied.

2. Equations of motion

We consider a CNT reinforced composite rectangular plate with length a, width b and total thickness h, which with four-edge simply-supported is subjected to the in-plane and transverse loads, as shown in Fig. 1. A Cartesian coordinate system is located in the middle surface of the CNT reinforced composite rectangular plate. The displacements of an arbitrary point within the plate are u, v and w in the x, y and z directions, respectively. The in-plane subjecting to the CNT reinforced composite rectangular plate load is expressed by $p = p_0 - p_1 \cos \Omega_2 t$, which is along the y direction at $x = a$. The transverse excitation subjecting to the CNT reinforced composite rectangular plate is assumed to be $F = F_0 - F_1 \cos \Omega_1 t$.

Two assumptions on the CNT composite plate are given as follows

1. The nanotubes have the same shapes which are uniformly distributed in the matrix.
2. The nanotubes have no agglomeration.

The carbon nanotubes are defined as the long hollow cylindrical fibers in the matrix while the composite material is the transversely isotropic material. Therefore, the stress–strain relationships of the tubes are given as follows

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} k_r + m_r & l_r & k_r - m_r & 0 & 0 & 0 \\ l_r & n_r & l_r & 0 & 0 & 0 \\ k_r - m_r & l_r & k_r + m_r & 0 & 0 & 0 \\ 0 & 0 & 0 & p_r & 0 & 0 \\ 0 & 0 & 0 & 0 & m_r & 0 \\ 0 & 0 & 0 & 0 & 0 & p_r \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{Bmatrix} \tag{1}$$

where k_r, l_r, m_r, n_r and p_r are Hill coefficients, specifically, k_r is the bulk modulus for the nanotubes, n_r is axial tensile modulus, l_r is transverse modulus, m_r and p_r are shear modulus in the perpendicular and are parallel to the surface of the nanotube direction, respectively [30].Based on the Mori–Tanaka method and the method of calculating the average stress for the composite material given in the article [31], the average strain $\bar{\boldsymbol{\varepsilon}}$ is defined as

$$\bar{\boldsymbol{\sigma}} = \mathbf{C}\bar{\boldsymbol{\varepsilon}}, \tag{2}$$

where $\mathbf{C}$ is average stiffness of the composite material.

Therefore, the average stress and strain of the material with two different components can be written as follows

$$\begin{aligned} &\boldsymbol{\sigma}_{\mathbf{m}} = \mathbf{C}_{\mathbf{m}} : \boldsymbol{\varepsilon}_{\mathbf{m}}, \quad \boldsymbol{\sigma}_{\mathbf{r}} = \mathbf{C}_{\mathbf{r}} : \boldsymbol{\varepsilon}_{\mathbf{r}}, \quad \bar{\boldsymbol{\sigma}} = \mathbf{c}_{\mathbf{m}}\boldsymbol{\sigma}_{\mathbf{m}} + \mathbf{c}_{\mathbf{r}}\boldsymbol{\sigma}_{\mathbf{r}}, \\ &\bar{\boldsymbol{\varepsilon}} = \mathbf{c}_{\mathbf{m}}\boldsymbol{\varepsilon}_{\mathbf{m}} + \mathbf{c}_{\mathbf{r}}\boldsymbol{\varepsilon}_{\mathbf{r}}, \end{aligned} \tag{3}$$

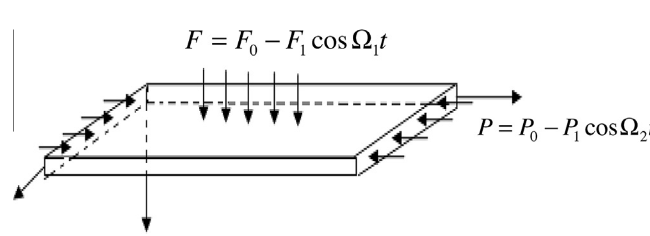

Fig. 1. The model of the CNT composite rectangular thin plate is given.