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Analytical Solution for Bending of Laminated Composites with Matrix Cracks

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ABSTRACT

An analytical, closed form solution is developed for balanced (not necessarily symmetric) laminates subjected to flexural deformation. The analytical solution provides spatial distribution of displacements and curvature, from which in-plane and intralaminar strains and stresses are obtained through differentiation and constitutive equations. The deformation is shown to consist of a homogeneous deformation plus perturbations near the crack tip. A methodology is proposed to separate the perturbation from the homogeneous deformation, to eliminate ill-conditioning of the eigenvalue/eigenvector problems that occurs otherwise. It is shown that, while the homogeneous deformation provides a macroscopic measure of damage in terms of reduced flexural stiffness of the laminate, the perturbation solution provides a detailed account of the intralaminar shear induced near the crack, which is used to calculate the extent of shear lag and the maximum intralaminar shear stress. The intact portion of the laminate is modeled without lumping it into a single equivalent lamina. Furthermore, laminas can be subdivided into multiple sub-laminates to increase the accuracy of the representation of intralaminar/interlaminar shear, which is shown to improve the predicted value of maximum interlaminar shear stress, which in turn is important for the prediction of matrix-crack induced delamination.

1. Introduction

Intralaminar matrix cracking is the first mode of damage in polymer-matrix laminated composites subjected to quasi-static, fatigue, and impact load. Matrix cracking increases the permeability of the laminate leading to gas/liquid leakage and facilitates access to contaminants that may degrade the fibers. Also, matrix cracking often precedes catastrophic modes of damage such as delamination, and fatigue life reduction. Furthermore, stiffness reduction of cracking laminas leads to stress redistribution to other laminas that may as a result fail in a catastrophic, fiber dominated mode. Therefore, prediction of damage initiation and evolution is an important ingredient of laminate failure prediction [1].

Matrix cracks are caused by a combination of transverse tensile and in-plane shear strain. Under these conditions, preexisting defects grow into cracks when the energy release rate exceeds the intralaminar fracture toughness of the lamina. Assuming linear elastic fracture mechanics [2] and periodicity leads to predictive methods that require the minimum number of material properties while achieving good comparisons with available experimental data. These solutions are either approximate, e.g., [3–7] or numerical, e.g., [8–11]. More refined methods require adjustable parameters, for example in the form of empirical hardening laws [12,13], and combinations of fracture and strength properties, for example [14].

The vast majority of analytical and semi-analytical solutions are restricted to symmetric laminates subjected to membrane loads only, and only matrix cracks are considered for the calculation of interlaminar stresses. To account for interaction between intralaminar cracks and delaminations, more complex numerical models such as reported in [15] are needed. Furthermore, most analytical and semi-analytical solutions assume either linear [16] or bi-linear [17] distribution of interlaminar stress through the undamaged sublaminates. Such limitation is removed in this work by subdividing each undamaged lamina into a sublaminate with as many layers as needed to achieve convergence in the value of the maximum interlaminar stress.

Matrix cracking of laminates subject to flexural deformation are analyzed in [18,19] using a clever analogy between laminates and orthotropic media. Such methodology was generalized in [20], but it relies on an "a priori" parametric study via finite element analysis (FEA) that restricts its applicability to those material systems

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included in the FEA study. A 1D beam bending model for $[0/90]_S$-like laminates where only one of the 90° laminas is allowed to crack is offered in [21,22]. The finite strip solution in [23–25] relies on the generalized plane strain assumption.

Both approximate and numerical solutions require either experiments or analytical solutions to validate them. Experiments are limited to a few laminate configurations and they are further limited in what can be measured. For example, stiffness reduction of carbon fiber laminates is very difficult to measure. Therefore, analytical solutions are desirable because they can be used as benchmarks, even if they are limited in scope of applicability to say, plane stress, and/or impose restrictions on the type of material behavior, such as say, elastically linear/damaging behavior.

In this work, a *closed form, analytical* solution is developed for balanced (not necessarily symmetric) laminates subjected to bending. Plane stress through the thickness and along one of the in-plane directions is assumed to reduce the problem to one dimension. The analytical solution provides spatial distribution of displacements and curvature, from which in-plane and intralaminar strains are obtained through differentiation, and stresses through constitutive equations. The solution is expressed as a combination of a fundamental solution and perturbation functions to represent the perturbation of the stress/strain field near the cracks. In this way, the near singularity of displacement-only solutions is removed. Furthermore, the perturbation terms lead naturally to computation of intralaminar/interlaminar stresses.

2. Approximations

The development of a closed form, analytical solution for bending of laminates with matrix cracks requires a number of approximations to reduce the 3D problem to 1D. These approximations are described as follows.

2.1. Fracture mechanics

Consider a thin, balanced laminate, with N laminas, subjected to bending load M_x only. All laminas are of the same material but oriented with respect to the x-axis in a laminate stacking sequence (LSS) such as $[0_m/90_n/\pm\theta_r]_S$, where $\theta < 45°$. Experimental evidence [26–33] indicate that in such laminates, the transverse laminas develop cracks as soon as the energy release rate in mode-I fracture G_I exceeds the intralaminar fracture toughness of the material G_{Ic}. Cracks start at defects within the transverse layer (90_n layer). Their propagation through the thickness of the ply is unstable [34, Section 7.2.1], reaching the interface suddenly. Upon further increase in applied load M_x or curvature κ_x, the thickness cracks grow again unstably parallel to the fiber direction, as illustrated in Fig. 1.

When subjected to bending deformation, only the laminas experiencing tensile stress may develop matrix cracks parallel to the fiber direction. If the cracked lamina spans across the midsurface, the thickness crack develops on the tensile side only. In this case, the transverse lamina is divided into two laminas, one cracked (tensile side) and another one virgin (compression side).

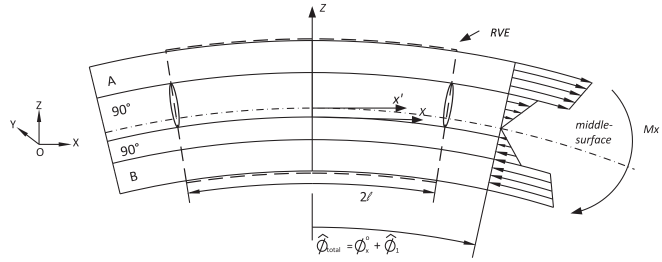

Fig. 1. Representative Volume Element (RVE) with dimensions $2\ell \times 1 \times 2h$, where $2h$ is the thickness of the laminate, and A, B, represent balanced sub-laminates.

Initially, cracks are not equally spaced but they become so as the crack density increases [35]. It is therefore possible to assume equally spaced cracks, which allows us to assume periodicity, and thus identify and use a representative volume element (RVE) to analyze this problem efficiently. The RVE encompasses the thickness of the laminate, a unit length along the fiber direction of the cracking lamina, and the arc length 2ℓ between existing cracks. The crack density in each lamina, denoted by subscript i, is defined as

$$\lambda_i = 1/2\ell \tag{1}$$

The analysis assumes that a very small crack density exists in the material, which may be justified as being representative of initial defects. An initial value $\lambda_i = 0.01\ \text{mm}^{-1}$ is used in the examples. New cracks are assumed to appear halfway between existing cracks, that is, as far as possible from the shear lag regions near existing cracks. The coordinate system for the RVE has its origin halfway between existing cracks, as shown in Fig. 1.

2.2. Plate kinematics

In this work, general laminates, such as $[0_m/90_n/\pm\theta_r]$, not necessarily symmetric, can be analyzed. Even if the undamaged (intact) laminate is initially symmetric, it will become unsymmetric when it cracks on the tensile side of the midsurface. Furthermore, bending deformation is antisymmetric with respect to the midsurface, with positive (negative) deformation above (below) the midsurface. In summary, due to antisymmetry of deformation and material properties with respect to the mid-surface of the laminate, all the laminate (with N laminas) needs to be analyzed. The following approximations are made:

I. Lines initially straight and normal to the mid-surface remain incompressible: $\epsilon_z \simeq 0$
II. A state of plane stress is assumed in the thickness direction, i.e., $\sigma_z^i = 0$
III. Due to intralaminar damage, a high order kinematics is needed to represent the beam deformation, and consequently lines initially straight and normal to the mid-surface are no longer straight and normal to the mid-surface. Furthermore, the deflection w^0 and the rotation ϕ_x^0 are not zero. Since the laminate is balanced, the deformation is symmetric with respect to the y–z plane (Fig. 1), and no bending is applied in the y-direction ($M_y = 0$), the intralaminar shear strain for each lamina can be written as the deviation from the average laminate rotation, as follows

$$\gamma_{xz}^i = \frac{\partial u^i}{\partial z} - \phi_x^0 \tag{2}$$

where $u_i(x,z)$ is the in-plane displacement of the i-lamina.

IV. In order to reduce the problem to 2D, a state of plane stress is assumed in the y-direction, i.e., $\sigma_y^i = 0$.
V. For a general laminate such as $[0_m/90_n/\pm\theta_r]$, the coupling terms D_{16} and D_{26} may be different from zero, but these terms decrease rapidly with increasing r. Therefore, each pair $(\pm\theta)$ is treated as an equivalent lamina without coupling ($Q_{16} = Q_{26} = 0$) and therefore $D_{16} = D_{26} = 0$ and $\gamma_{xy}^i = 0$.
VI. Since cracks appear equally spaced on both sides of the y–z plane (Fig. 1), the domain is symmetric with respect to the y–z plane and $\gamma_{yz}^i = 0$.