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Free vibration of two-directional functionally graded beams

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ABSTRACT

This paper presents a theoretical investigation in free vibration of a functionally graded beam which has variable material properties along the beam length and thickness. It is assumed that material properties vary through the length according to a simple power law distribution with an arbitrary power index and have an exponential gradation along the beam thickness. The characteristic equations are derived in closed form. The governing equation can analytically reduce to the classical forms of Euler–Bernoulli beams if the gradient index disappears. Analytical solutions of the natural frequencies are obtained for graded beams with clamped-free and hinged–hinged end supports. Results show that the variations of material properties in the beam length and thickness have a strong influence on the natural frequencies. It is also shown that there exists a critical frequency depending on the gradient parameter. The natural frequencies have an abrupt jump when across its critical frequencies. The derived results can be useful for designing non-homogeneous beams which may be required to vibrate with a particular frequency.

1. Introduction

Functionally graded (FG) materials are one class of non-homogeneous materials with a gradual transition between two or more phases, which are used in many civil, mechanical and aerospace engineering structures. As the use of FG materials increases, FG materials may be fabricated into various structures including beams [1,2], plates [3], microshells [4], nanobeams [5] and so on. For FG beams, a review of the literature reveals that the previous works have consider the gradation of the material properties in the thickness direction or in the axial direction.

For FG beams or plates with the gradation of the material properties in the thickness direction, there have been a large number of researches devoted to stress, deformation, stability and vibration [5–26]. Akgoz and Civalek [5] used implementing the minimum total potential energy principle and Navier solution procedure to present a shear deformation beam model for FG microbeams with new shear correction factors. Aydin [6] studied free vibration of FG beams with exponential gradation along the beam thickness. Giunta et al. [7] conducted free vibration analysis of FG beams with a power law gradation on the cross-section by hierarchical theories. Li et al. [8] used classical and first-order shear deformation beam theories to investigate free vibration of FG beams. Li et al. [9] presented a size-dependent FG piezoelectric beam model by a variational formulation. Liu and Shu [10] developed an analytical solution to study the free vibration of exponential FG beams with a single delamination. Li presented [11] a new unified approach to study the static and dynamic behaviors of FG beams with the rotary inertia and shear deformation included. Mahi et al. [12] presented an analytical method to study the free vibration of symmetric FG beams. Mohanty et al. [13] used finite element method to studied the dynamic stability of FG beams and FG sandwich beams. Pandey and Pradyumna presented [14] a layerwise finite element formulation to investigate free vibration of FG sandwich plates in thermal environment. Sankar and Tzeng [15] obtained exact solutions for thermal stress distributions in a FG beam with an exponential variation of material properties through the thickness. Simsek and Kocaturk [16] investigated free vibration characteristics and the dynamic behavior of a FG simply supported beam under a concentrated moving harmonic load. Sina et al. [17] presented a new beam theory different from the traditional first-order shear deformation beam theory to analyze free vibration of FG beams. Trung-Kien et al. [18] proposed a new higher-order shear deformation theory to study buckling and free vibration analysis of isotropic and FG sandwich beams. Wei et al. [19] proposed an analytical method to study free vibration behiviors of FG beams with edge cracks. Yang et al. [20] used a meshfree boundary domain integral equation method to investigate free vibration of the FG sandwich beams. Anandrao et al. [21] investigated free vibration of FG beams with various classical boundary conditions by two separate finite element formulations. One based on Euler–Bernoulli beam theory and the other based on Timoshenko beam theory are developed. Asghari [22] presented a

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size-dependent formulation for Timoshenko beams made of a functionally graded material. Except the above studies, the fracture behavior of FG structures was investigated by using the extended finite element method [23–26].

For axially FG beams, the similar problem becomes more complicated because of the governing differential equation with variable coefficients. So far, a relatively few researchers have considered the gradation of the material properties in the axial direction [27–33]. Alshorbagy et al. [27] used numerical finite element method to study the dynamic characteristics of functionally graded beam with material graduation in the axial direction based on the power law. Huang and Li [28] presented a novel and simple approach to solve natural frequencies of free vibration of an axially graded and non-uniform beam. Sarkar and Ganguli [29] studied the free vibration of axially FG Timoshenko beams with polynomial gradation of the material mass density, elastic modulus and shear modulus, along the length of the beam. Shahba and Rajasekaran [30] studied the free vibration and stability of axially FG tapered Euler Bernoulli beams through solving the governing differential equations of motion. Şimşek et al. [31] used Euler Bernoulli beam theory to investigate linear dynamic behavior of an axially FG beam with simply supported edges. Li et al. [32] studied free vibration of exponentially FG beams and obtained exact frequency equations of the free vibration problem. Kukla and Rychlewska [33] studied free vibration of axially FG beams consisting of two segments.

The previous works focused on the dynamic characteristics of FG beams with material graduation in the length direction (as shown in Fig. 1(a)) or in the thickness direction (as shown in Fig. 1(b)). However, there are practical occasions which require the materials graded in two or three directions. So, it is necessary to develop appropriate methods to investigate the dynamic behaviors of multi-directional functionally graded structures. But, due to the complexity of the problem caused by the multi-directional inhomogeneity, it is difficult to obtain the exact solution. Nie et al. [34] used a semi-analytical numerical method to study the dynamic behavior of multi-directional functionally graded annular plates. The objective of this paper is to present an analytical method to investigate the free vibration of FG beams with the gradation of the material properties along the beam length and thickness (as shown in Fig. 1(c)). First, based on Euler–Bernoulli beam theory, the vibration problem of FG beams is turned into a governing differential equation. Second, the natural frequencies and mode shapes can be determined by solving the governing differential equation. Finally, the influences of material gradation parameters on vibration characteristics of FG beams are investigated in detail.

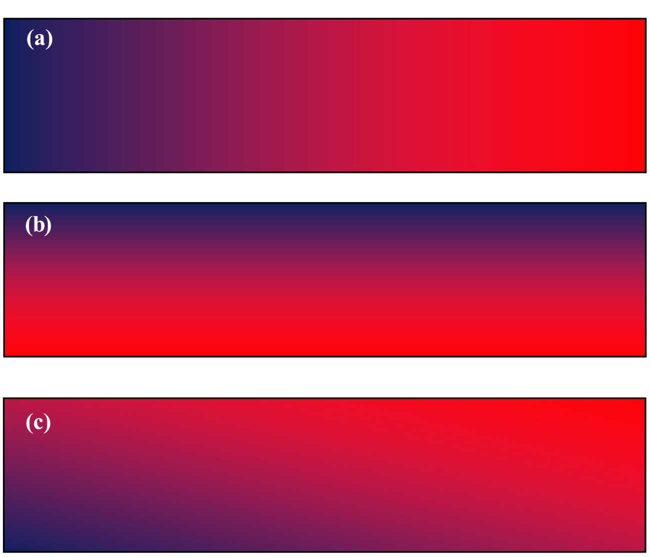

Fig. 1. Schematic of functionally graded beams: (a) an axially graded beam, (b) a thickness graded beam and (c) a two-directional graded beam.

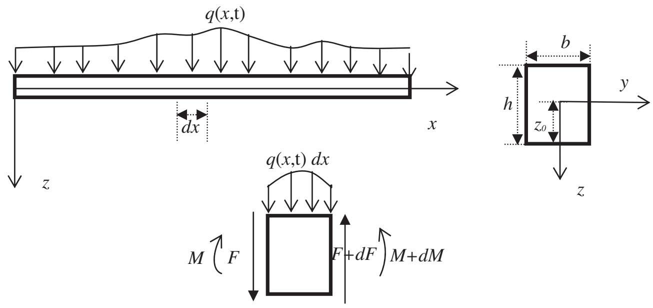

Fig. 2. Schematic of FG beam.

2. Mathematical formulations

2.1. Governing differential equation

Consider a straight and uniform cross-section FG beam with the length l, the width b, the depth h, and a rectangular cross section (see Fig. 2). It is assumed that the material properties vary continuously along the length and thickness direction. In the present work, Young's modulus E and mass density ρ are assumed to vary in the x- and z-direction according to the following expressions

$$E = E_0 \exp(\beta x) f(z) \quad \rho = \rho_0 \exp(\beta x) g(z) \tag{1}$$

where E_0 and ρ_0 are constants of Young's modulus and mass density, respectively. β is a constant characterizing the gradual variation of the material properties along x-direction. $f(z)$ and $g(z)$ are the functions of variable z.

According to Euler–Bernoulli beam theory, the following assumptions are made: (1) All the cross-section of FG beams remain plane after deformation. However, they can undergo a rigid body displacement in x–z plane and also a rotation about y-axis. (2) The effects of rotary inertia and shear deformation in x–z plane can be ignored. (3) The angle of rotation is small so that the small angle assumption can be used.

Based on these assumptions, the axial and the bending rotation displacement fields are respectively given as

$$u = -\theta z \tag{2}$$

$$\theta = \frac{\partial w(x,t)}{\partial x} \tag{3}$$

where x and z are the spatial coordinates as shown in Fig. 2, t is time, w is the transverse displacement.

Considering the small deformations and assuming the material of FG beams obeys Hooke's law, the strains and stresses in the beam are respectively obtained using Eqs. (1) and (2) as,

$$\varepsilon_x = \frac{\partial u}{\partial x} = -z\frac{\partial^2 w(x,t)}{\partial^2 x} \tag{4}$$

$$\sigma_x = E\varepsilon_x = -Ez\frac{\partial^2 w(x,t)}{\partial^2 x} = -E_0 \exp(\beta x) f(z) z \frac{\partial^2 w(x,t)}{\partial^2 x} \tag{5}$$

According to Eq. (5), the bending moment M is given by

$$M = b\int_{z_0-h}^{z_0} \sigma_x z dz = -DE_0 \exp(\beta x)\frac{\partial^2 w(x,t)}{\partial^2 x} \tag{6}$$