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Stochastic free vibration analysis of laminated composite plates using polynomial correlated function expansion

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ABSTRACT

Over the past few decades, composite materials are extensively used in various industries due to their high specific stiffness, strength, weight sensitivity and cost-effectiveness. However, lack of complete control over the manufacturing process results in undesirable uncertainties, which in turn affect the vibrational characteristics of systems. This paper presents a novel approach, referred to as polynomial correlated function expansion (PCFE), for stochastic free vibration analysis of composite laminate. The proposed approach facilitates a systematic mapping between the input and output variables by expressing the output in a hierarchical order of component functions. The component functions are expressed in terms of extended bases and the unknown coefficients associated with the bases are determined by employing homotopy algorithm. The proposed approach has been employed for stochastic free vibration analysis of laminated composite plates. Results obtained using PCFE have been compared with results obtained using radial basis function (RBF) and conventional response surface method (RSM). Compared to RBF and RSM, PCFE yield more accurate result from considerably fewer sample points. Furthermore, case studies with different ply orientations have also been performed. Based on the numerical results, new physical insights have been drawn on dynamic behaviour of composite laminates.

1. Introduction

The fiber reinforced plastic (FRP) composite materials are being increasingly used in aerospace [1,2], automotive [3,4], marine [5,6] and other engineering applications [7–9] for many years because of their superior strength and stiffness to weight ratios, corrosion resistance, structural tailoring capability *etc.* For the FRP composite laminates, the processes of lay-up and curing are relatively complex, involving more structural and material complications in contrast to conventional isotropic materials. Due to lack of complete control over the manufacturing, uncertainties arise in geometrical parameters and material properties, especially during hand layup procedure. Some of the main reasons for composite material property variations are lack of resin, excess resin between the layers, air entrapment, incomplete curing of resin, delamination and number of geometrical parameters such as fiber alignments, volume fractions, voids and others. The presence of these uncertainties may have substantial effects on the fundamental structural response such as natural frequencies and mode shapes. It is, therefore, important to incorporate uncertainties arising due to randomness in material properties and structural configuration.

The most popular method for quantifying uncertainty is the Monte Carlo simulation (MCS) [10,11]. This method involves deterministic evaluation of response at randomly generated samples and in most cases is straightforward to employ. A number of improvements to MCS (*e.g.,* Latin Hypercube sampling [12,13], stratified sampling [14,15] *etc.*) have also been proposed. All these methods can be grouped as statistical approach (SA). However, SA is computationally expensive, specifically for problems that are already complicated in deterministic state. Thus, use of these methods is limited to verification of newly developed methods only.

An advantageous alternative of the SA is the non-statistical approach (NSA). Within the framework of NSA, one first determine the responses at preselected samples, often termed as design of experiments [16,17]. Next, a functional relationship is generated by mapping the input and output variables. This functional relationship is the backbone of NSA. NSA that are popular among researchers include but are not limited to polynomial chaos expansion (PCE) [18–21], Kriging [22–24], radial basis function (RBF) [25,26], conventional response surface method (RSM) [27,28], moving least square [29,30] and high dimensional model representation [31–33].

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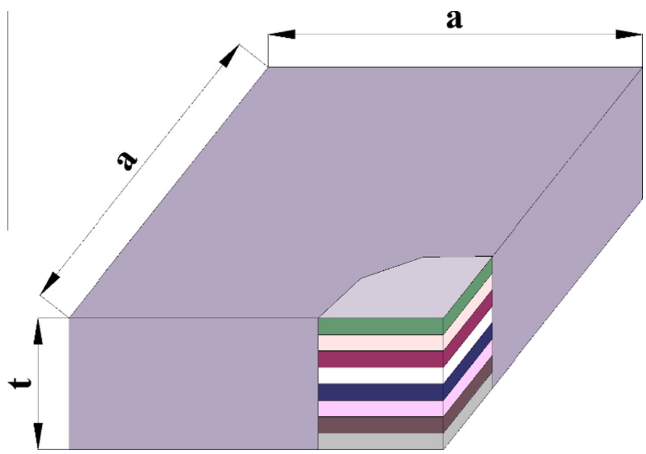

Fig. 1. Schematic diagram of laminated composite plate (not to scale) is having eight lamina. All the edges of the plate are considered to be clamped.

This paper introduces a novel approach, referred to as polynomial correlated function expansion (PCFE) [34,35], for stochastic free vibration analysis of composite plates. Compared to popular NSA, PCFE has certain desirable properties. Firstly, PCFE is convergent in mean square sense because the unknown coefficients associated with the bases are determined by minimizing the $\mathcal{L}^2$ error norm. Secondly, PCFE is a finite series consisting of 2^N component functions, where N is the number of random variables. Thus, if the component functions are convergent, PCFE provides an exact solution. Thirdly, PCFE is optimal in Fourier sense. This is because the hierarchical orthogonality of the component function is satisfied while determining the unknown coefficients. Finally, PCFE provides a common platform for dealing with both dependent and independent random variables without the need of any *ad hoc* transformations.

The main objective of this paper is to perform stochastic free vibration analysis of laminated composite plates. The plate under consideration consists of eight lamina as shown in Fig. 1. All the edges are considered to be clamped. The material properties (density, elastic modulus, Poisson's ratio and shear modulus), thickness and ply orientations are considered to be random. As a consequence, the system is having twenty-three random variables. The study has been performed for four ply orientations, namely (1) cross-ply (symmetric and anti-symmetric), (2) angle-ply (symmetric and anti-symmetric), (3) quasi-isotropic ply and (4) mixed ply. Results obtained have been benchmarked against MCS solutions and other popular NSA.

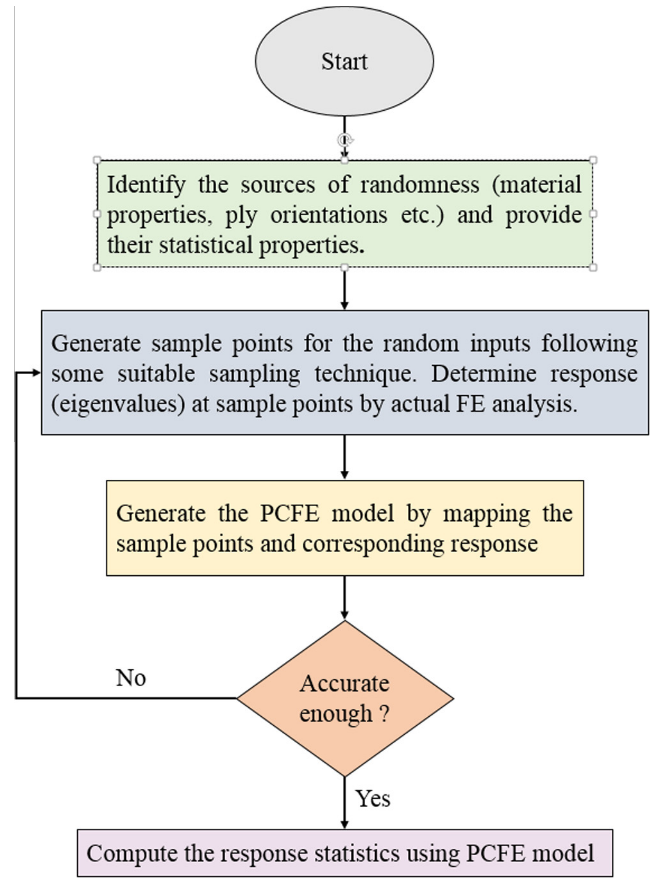

Fig. 2. Flow chart for PCFE.

The rest of the paper is organised as follows. After providing a brief description of the finite element (FE) formulation in Section 2, Section 3 describes the fundamentals of PCFE. Section 4 provides a generalised framework for stochastic free vibration analysis of laminated composite plate using PCFE. The results obtained are presented in Section 5. Finally, Section 6 provides the concluding remarks.

Table 1
Description of random variables [43].

Sl.	Variable	Description	Type	Mean	SD
1	E_{11} (MPa)	Elastic modulus along longitudinal direction	Lognormal	4.2×10^4	1512
2	E_{22} (MPa)	Elastic modulus along lateral direction	Lognormal	1.13×10^4	621.5
3	G_{12} (MPa)	Shear modulus	Lognormal	4.5×10^3	189
4	G_{13} (MPa)	Shear modulus	Lognormal	4.5×10^3	189
5	G_{23} (MPa)	Shear modulus	Lognormal	4×10^3	168
6	ν_{12}	Poisson's ratio	Lognormal	0.3	0.0042
7	d_1 (mm)	Thickness	Rayleigh	0.45	0.058
8	d_2 (mm)	Thickness	Rayleigh	0.45	0.058
9	d_3 (mm)	Thickness	Rayleigh	0.45	0.058
10	d_4 (mm)	Thickness	Rayleigh	0.45	0.058
11	d_5 (mm)	Thickness	Rayleigh	0.45	0.058
12	d_6 (mm)	Thickness	Rayleigh	0.45	0.058
13	d_7 (mm)	Thickness	Rayleigh	0.45	0.058
14	d_8 (mm)	Thickness	Rayleigh	0.45	0.058
15	$\rho(t/mm^3)$	Density	Lognormal	1.9×10^{-9}	1.9×10^{-11}
16–23	$\theta_1 - \theta_8$	Orientation angle*	Uniform	–	1.732

* Mean of orientation angle is case specific. The details are provided in Section 5.