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Multi-objective free-form optimization for shape and thickness of shell structures with composite materials

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ABSTRACT

In this paper, we present a two-phase optimization method for designing the shape and thickness of shell structures consisting of orthotropic materials. We consider a multi-objective in terms of the compliances under multi boundary conditions, and use the weighted sum compliance as the objective functional and minimize it under the volume and the state equation constraints. The 1st phase is shape optimization, in which a shell structure is varied in the out-of-plane direction to the surface to create its optimal shape. In the 2nd phase, thickness optimization is implemented after shape optimization to decrease the compliance further. A free-form shape and thickness optimization problem is formulated in a distributed-parameter system based on the variational method. The shape and thickness sensitivities are theoretically derived and applied to the H^1 gradient method for shape and size optimization. The optimal multi-objective free-form of a shell structure with an orthotropic material can be determined using the proposal method, and the influence of the orthotropic angle to the optimal shape and thickness distribution is investigated in detail.

1. Introduction

Shell structures are widely used in various industrial products. From an economic point of view, weight reduction is strictly required in the structural design of cars, aircrafts and so on. The usage of composite materials in shell structures is one of the solutions to meet the requirement since they have higher material performances than metals. In especial, orthotropic materials can be used for making specific stiff directions of shell structures. Moreover, with design optimization, mechanical properties can be significantly improved.

In design optimization of shell structures with orthotropic materials, three structural optimization methods, topology optimization, shape optimization and size optimization, are commonly considered among designers. Most of the previous works focused on the topology optimization of shell structures with composite materials. Matteo and Pierre [1] studied topology optimization for minimizing the weight of structures under compliance and stress constraints. The investigation of material influence has also been demonstrated by several authors. Tenek and Hagiwara [2] investigated the isotropic and anisotropic influence of plates for minimizing their weight. Blaspues and Stolpe [3,4] treated orthotropic beams with several material distribution patterns to find the influence of material characteristics. Furthermore, size optimization for composite materials has also been reported by some scholars. Li and Yoshihiro [5] designed optimal laminated shallow shell structures under variable material orientations. Multi-objective optimization of composite laminate was proposed by Nik et al. [6]. It was found that by allowing the fiber orientation perpendicular to the loading direction, both stiffness and buckling behavior under volume constraint can be improved and the Pareto optimal design space can be found using the genetic algorithm. Zhu et al. [7] presented a multi-objective optimization combining weight, thermal expansion and buckling load by varying ply thickness of CFRP beams using the genetic algorithm.

Shape optimization of shell structures, including parametric and non-parametric methods, is also an effective means in optimal design for shell structures under the material determination. Most of the previous works of shape optimization design of shell structures are classified as parametric methods, in which a shell structure is parameterized using CAD parameters such as height and thickness, parametric surface or design elements in advance. The parametric method is effective for reducing the number of design variables and seldom causes a jagged surface problem. However, designers need considerable knowledge and experience

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of shape parameterization. The parametric methods have been studied not only for designing simple formed shell structures but also for arbitrarily formed shell structures. Ramm et al. [8] introduced the Bezier function and used its control points as the design variables. Rao and Hinton [9] used the Coons patch in shape optimization. Ugail and Wilson [10] obtained a smooth shell surface with the partial differential equation method. In contrast, non-parametric techniques for designing free-form shell structures were also proposed previously. Daoud et al. [11] proposed a non-parametric method with an artificial filtering technique for maintaining smoothness. Leiva [12] proposed a non-parametric method based on the basis vector method, but without smoothing. Some other works using non-parametric methods that only dealt with shape sensitivity analysis of shell structures were also carried out [13–15].

The size optimization involving thickness optimization methods have been developed by several authors. For example, Tomás and Martí [16] introduced shape and size optimization of concrete shell structures used control points as the design variables. Velea at el. [17] considered thickness of the FRP structures by a combined method. The above two methods, are classified as parametric method which needs designers' considerable knowledge and experience of thickness parameterization. On the other hand, non-parametric methods for thickness optimization have been also developed by some scholars. Arnout at el. [18] proposed a non-parametric method for thickness optimization considering stress with Kreisselmeier–Steinhauser function. Czarnecki and Lewinski [19] proposed a non-parametric method based on a potential method. Both of the above two works used the finite dimensional approach, however, could not assure smoothness of structures.

The free-form optimization method for shells is one of the non-parametric methods for arbitrarily formed shell structures that can determine the optimal smooth and natural free-form without causing jagged surfaces and without requiring shape parameterization. This method was proposed based on the traction method or H^1 gradient method [20–23]. However, there has seldom study of shape optimization for shell structures consisting of orthotropic materials. In our previous work [23], we developed a free-form optimization method for determining free-form optimal shell structures with isotropic material. In the present work, the method is extended to a shell structure with an orthotropic material, and the influence of the orthotropic angle is investigated. Moreover, in this study, a non-parametric method for thickness distribution based on a gradient method is newly developed introducing Poisson's equation both to reduce the objective functional and maintain thickness smoothness. That is, the shape and thickness optimization method is integrated to obtain higher stiffness of shell structures, or to eliminate waste of the material.

The key point of the integration of the two-phase optimization of shell structures is determining the shape at first, and reducing unnecessary thickness for lighting-weight subsequently. In addition, shell structures with multi boundary conditions are considered for actual applications. The term of multi boundary conditions here refer to more than two boundary conditions that act independently on a shell structure. In the present work, we use the weighted sum compliance under multi boundary conditions as the objective functional. The compliance minimization problem is formulated in a distributed-parameter shape and thickness optimization system. The sensitivity functions, also called the shape and thickness gradient functions or the optimal conditions, are theoretically derived using the material derivative method and the adjoint method. The derived shape gradient functions are applied to the proposed two-phase optimization method. In the following sections, the governing equation for shell structures, the formulation of the compliance minimization problem, the two-phase optimization method and the H^1 gradient methods for shape and thickness optimization are described in detail. At last, numerical analysis results for several design problems are presented to demonstrate the effectiveness and practical utility of our solution. In addition, the influence of the orthotropic angle is investigated by changing the material direction of the initial shape.

2. Governing equation for a shell structure with an orthotropic material

As shown in Fig. 1(a) and Eqs. (1)–(3), we consider a shell structure owning an initial bounded domain $\Omega \subset \mathbb{R}^3$ with the boundary $\partial\Omega$, mid-surface A (with the boundary ∂A), side surface S and thickness h. It is assumed for simplicity that a shell structure occupying a bounded domain is a set of infinitesimal flat surfaces. The notation dA expresses a small area.

$$\Omega = \left\{ \{(x_1, x_2, x_3) \in \mathbb{R}^3 | (x_1, x_2) \in A \subset \mathbb{R}^2, \ x_3 \in \left(-\frac{t}{2}, \frac{t}{2}\right) \right\} \tag{1}$$

$$\Omega = A \times \left(-\frac{t}{2}, \frac{t}{2}\right) \tag{2}$$

$$S = \partial A \times \left(-\frac{t}{2}, \frac{t}{2}\right) \tag{3}$$

It is assumed that the mapping of the local coordinate system $(x_1, x_2, 0)$, which gives the position of the mid-surface of the shell structure, to the global coordinate system (X_1, X_2, X_3), i.e., $\phi : (x_1, x_2, 0) \in \mathbb{R}^3 \mapsto (X_1, X_2, X_3) \in \mathbb{R}^3$, is piecewise smooth. The Mindlin–Reissner plate theory is applied concerning bending of plates. The coupling of the membrane stiffness and bending stiffness is ignored for simplicity. Using the sign convention in Fig. 1 (b), the displacements expressed by the local coordinates $\boldsymbol{u} = \{u_i\}_{i=1,2,3}$ are considered by dividing them into the displacements in the in-plane direction $\{u_\alpha\}_{\alpha=1,2}$ and in the out-of-plane direction u_3, shown as:

$$u_\alpha(x_1, x_2, x_3) = u_{0\alpha}(x_1, x_2) - x_3\theta_\alpha(x_1, x_2), \tag{4}$$

$$u_3(x_1, x_2, x_3) = w(x_1, x_2). \tag{5}$$

The notations $\{u_{0\alpha}\}_{\alpha=1,2}$, w and $\{\theta_\alpha\}_{\alpha=1,2}$ express the in-plane displacements, out-of-plane displacement and rotational angles of the mid-surface of the shell structure, respectively.

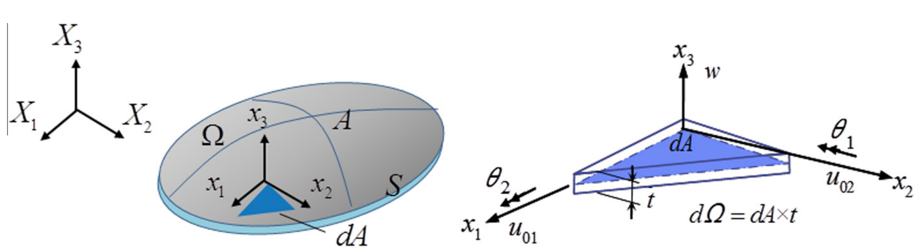

Fig. 1. Shell as a set of infinitesimal flat surfaces. (a) Geometry of shell and global coordinates and (b) Local coordinates and DOF of flat surface.