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Design optimization of foam-reinforced corrugated sandwich beams

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ABSTRACT

A combined analytical and numerical study is carried out for the structural stiffness, collapse strength and minimum mass design of foam-filled corrugated sandwich beams under transverse three-point bending. Both close-celled aluminum foam and polymer foam as the filling material are considered. Based upon a micromechanics-based model, effective elastic constants of foam-filled corrugations are derived using the homogenization method. To analytically predict the initial collapse strength, six different failure modes are considered, with the effect of loading platen width accounted for. Finite element simulations are performed to validate the analytical predictions, with good agreement achieved. Minimum mass design is obtained as a function of structural strength, and the influence of foam material and loading platen width is quantified. The structural efficiency of foam filling to reinforce the sandwich is assessed on the basis of equal mass and the underlying mechanisms explored. It is shown that polymer foam-filled corrugations are more weight efficient than unfilled ones of equal mass.

1. Introduction

By mingling the advantageous attributes of stochastic foams and periodic lattices, hybrid-cored sandwich structures can be constructed for high stiffness, strength and energy absorption. Such hybrid sandwich cores include pin-reinforced foams [1–4], polymer foam-filled polymer lattices [5,6], polymer foam-filled fibre reinforced composite lattices [7,8], polymer foam-filled metallic lattices [9,10], and metallic foam-filled metallic lattices [11–13]. Existing studies focused mainly on the out-of-plane compression behavior of these novel structures, while only a few concerned about their bending performance [14–17].

Among the hybrid foam-lattice cores studied thus far, metallic corrugations filled with either polymer or metallic foams received special attention on account of their relatively low manufacturing cost and good structural performance. Vaziri et al. [9] found numerically that sandwich plates with polymer foam-filled metallic corrugated cores can perform as well, or nearly as well, as plates of the same weight with unfilled cores in terms of: (i) basic core responses to crush, shear and stretch, (ii) clamped plate response to quasi-static punch load, and (iii) plate response to impulsive load. Using a combined experimental and theoretical study, Yan et al. [11] and Han et al. [12] studied the quasi-static uniaxial compression behavior of metallic corrugated sandwiches filled with close-celled aluminum foams, while Yu et al. [13] explored the corresponding dynamic crushing responses. It is demonstrated [11–13] that foam filling can dramatically increase the specific compression strength and specific energy absorption of corrugated sandwich cores. Subsequently, based upon mainly experimental measurements, Yan et al. [17] studied the transverse three-point bending (i.e., bending plane normal to corrugation axis, as shown in Fig. 1) behaviors of metallic corrugated sandwich beams filled with aluminum foams. While filling of aluminum foam led to dramatically increased bending stiffness and strength of the sandwich, its mass also increased considerably [17]. Under transverse three-point bending, it is uncertain whether filling a corrugated core with foam can result in a larger failure load than that of an empty core with equal mass.

As a companion study of Yan et al. [17], this article explores further the concept of foam filling to reinforce corrugated sandwiches in three-point bending, with emphasis placed upon minimum mass design, structural efficiency assessment and strengthening mechanisms analysis. In particular, different from the Yan et al. [17], the benefit of foam filling is assessed on the basis of equal mass and both aluminum and polymer foams having high porosities are considered as the filling material.

This study firstly derives analytical expressions of both the stiffness and initial collapse strength for foam-reinforced corrugated

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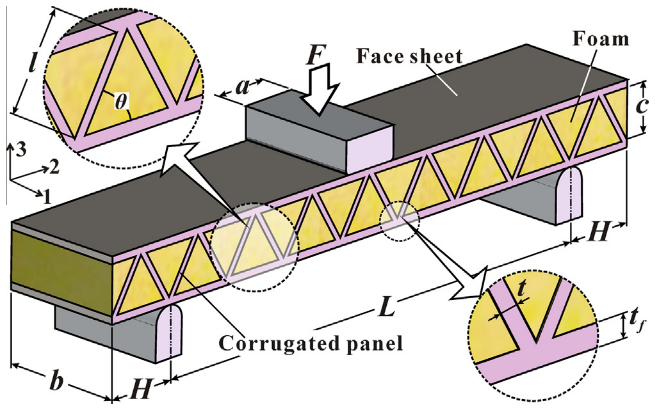

Fig. 1. Foam-reinforced corrugated sandwich beam in transverse 3-point bending.

sandwich beams in transverse three-point bending. Subsequently, minimum mass designs are carried out to quantify the structural efficiency of foam-filled corrugations on the basis of equal mass. The underlying strengthening mechanisms are explored and collapse mechanism maps constructed. Finally, finite element (FE) simulations are performed to visualize the failure modes of different foam-corrugation combinations and to validate the analytical model predictions.

2. Stiffness and strength of foam-filled corrugated sandwich beams

Consider a foam-filled corrugated sandwich beam subjected to transverse 3-point bending with span length L and overhang H, as depicted in Fig. 1. Relevant geometric variables are: face sheet thickness t_f, corrugated member thickness t, corrugated member length l, corrugation angle θ, core height c, and sandwich beam width b. The volume fraction λ of the corrugated members and density ρ_c of the unit cell in the sandwich core are given by:

$$\rho_c = \lambda\rho_s + (1-\lambda)\rho_f \tag{1}$$

$$\lambda = \frac{2t}{l\sin 2\theta} \tag{2}$$

where ρ_s and ρ_f represent the density of the corrugated member material and foam, respectively. Let E_s and v denote the Young's modulus and Poisson ratio of the corrugated member material; while E_f and v_f denote the Young's modulus and Poisson ratio of the foam.

It is assumed that the corrugated members are perfectly bonded to the face sheets and no sliding occurs when subjected to loading. It is further assumed that the corrugated members and the filling foam keep close contact with each other during deformation, even though slip may occur at the interface.

2.1. Effective elastic constants of foam-filled corrugated core

To analyze the structural response of the sandwich beam under 3-point bending, the effective elastic constants of its foam-filled core are obtained with the homogenization method. For periodic lattice cores, the homogenization method has been widely used to calculate their equivalent elastic structural performance [14,18–20]. Using this approach, Liu et al. [19,20] derived the effective stiffness matrix of empty (unfilled) corrugated cores.

The foam-filled corrugated core may be analyzed at two different scales: (a) at the macroscale, it is treated as a homogeneous continuum solid; (b) at the microscale, the foam fillers and the corrugated members are separately considered. The derivation of micro–macro relations for such a periodic medium relies on the analysis of its representative volume element (RVE, or unit cell).

2.1.1. Homogenization of foam-filled corrugated core

As schematically shown in Fig. 2(a), when subjected to a $\bar{y}-\bar{z}$ plane macroscopic strain $\mathbf{E}$, the corrugated member may be characterized as an Euler–Bernoulli beam of unit width (along the $\bar{x}$-direction), clamped at both ends, since it is typically 100–1000 times stiffer than the foam filler. For a unit cell containing two corrugated beam members surrounded by foam filling, analogous to the analysis of pin-reinforced foam cores [14], its macroscopic strain energy density may be written as:

$$G = G_b + G_f \tag{3}$$

$$G_b = \frac{1}{\Omega}\sum_{i=1}^{2}\left[\frac{1}{2}\left(\tilde{\mathbf{u}}^{(i)} + 2\tilde{\mathbf{u}}_p^{(i)}\right)^T \tilde{\mathbf{K}}^{(i)}\tilde{\mathbf{u}}^{(i)} - \tilde{g}_p^{(i)}\right] \tag{4}$$

$$G_f = (1-\lambda)\left(\frac{1}{2}C^f_{hjkl}E_{hj}E_{kl}\right) + \frac{1}{\Omega}\sum_{i=1}^{2}\tilde{g}_p^{(i)} \tag{5}$$

where G_b and G_f are the strain energy of the beam members and foam filler, respectively, Ω represents the current volume of the unit cell, superscript/subscript f denotes the foam, and $\tilde{\mathbf{u}}^{(i)}$ is the global nodal displacement vector for the ith inclined beam characterized by end nodes ζ and τ (Fig. 2(b)):

$$\tilde{\mathbf{u}}^{(i)} = \mathbf{T}^T\tilde{\mathbf{u}}^{(i)e} \tag{6}$$

$$\tilde{\mathbf{u}}^{(i)e} = [w_\zeta, v_\zeta, \theta_{\zeta x}, w_\tau, v_\tau, \theta_{\tau x}]^{(i)T} \tag{7}$$

Here, $\tilde{\mathbf{u}}^{(i)e}$ is the nodal displacement vector under local coordinates (y, z), $\mathbf{T}$ is the transformation matrix between local and global coordinates (see Appendix A), and e denotes values in local coordinates. The global nodal displacement vector for the ith beam may be written as:

$$\tilde{\mathbf{u}}^{(i)} = [\Delta_1, \Delta_1, 0, 0, 0, 0]^{(i)T} \tag{8}$$

where Δ_1 and Δ_2 denote the projections of displacements $\mathbf{\Delta}$ of the end nodes of the inclined beam (Fig. 2(a)), given by:

$$\mathbf{\Delta} = \frac{c}{\sin\theta}\mathbf{E}\mathbf{N}_0 \tag{9}$$

with

$$\mathbf{E} = \begin{bmatrix} E_{22} & E_{23} \\ sym & E_{33} \end{bmatrix}, \quad \mathbf{\Delta} = (\Delta_1\mathbf{m}, \Delta_2\mathbf{r})^T. \tag{10}$$

Here, $\mathbf{N}_0$ is the unit vector along which the beam member is initially aligned.

In Eq. (4), $\tilde{\mathbf{u}}_p^{(i)}$ is the nodal displacement vector of the ith beam induced by lateral normal stress $p^{(i)}$ (Fig. 2(b)), which represents the coupling effect between the beam and foam:

$$\tilde{\mathbf{u}}_p^{(i)} = \mathbf{T}^T\tilde{\mathbf{u}}_p^{(i)e} \tag{11}$$

$$\tilde{\mathbf{u}}_p^{(i)} = \left[\frac{vp^{(i)}}{E_s}l \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\right]^T \tag{12}$$

The effect of shear stress on the lateral surface of the beam is ignored. The strain energy contributed by lateral normal stress $p^{(i)}$ may be divided into two parts: one is related to elongation of the ith beam, as calculated in Eqs. (11) and (12); the other, related to compression on beam lateral surface and represented by $\tilde{g}_p^{(i)}$ in Eqs. (4) and (5), is eliminated during summation of the total strain energy in Eq. (3).

The macroscopic deformation of the foam in a unit cell is approximately equal to that of the unit cell, as shown in Eq. (5). Therefore, with close contact assumed between beam members